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ACTION OF THE SUN ON THE EARTH ROTATION

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1. Introduction. Now the warming of climate and various catastrophic influences of the natural phenomena on the human society life is observed. The changes of climate were also observed in the past: periodic changes of sedimentary layers on the continents and at the oceans, fluctuations of oceans levels and repeating traces of activity of glaciers are observed. The climate of the Earth directly depends on insolation, on which, in turn, the orbital and rotation of the Earth influences. Such Astronomical theory of glacial ages Milutin Milankovich has developed in 20th years of XX century, and then it was considered by some groups of researchers. At that the problem of rotation was solved rather approximately. It can be fully solved by numerical method. To consider all features it is necessary to analyse the bases at a conclusion of the differential equations. Besides a number of the physical reasons of dynamics of the Earth axis follows from the theorem of the moments. Therefore in the beginning we shall consider its consequences on an example of the three bodies.

2. The theorem of the moments and its approached results. Rotation of the Earth it is considered in non-rotative system of coordinates $x_1y_1z_1$, connected with the center of mass of the Earth. Rotation of mechanical system is described by the theorem of change of the angular momentum relatively the moving center:

$$\frac{dK_O}{dt} = \sum \vec{m}_O(\vec{F}_k)\,, \qquad (1)$$

where $\vec{K}_O$ - the angular momentum of the Earth relatively the center O in the system $x_1y_1z_1$; $\vec{m}_O(\vec{F}_k)$ - the moment relatively this center of external force $\vec{F}_k$ acting on a body in the same system $x_1y_1z_1$.

Let whirligig (see fig.1*a*) forms an angle θ with an axis z_1 and rotates around the axis z with angular velocity $\dot{\varphi}$. The force P is enclosed in its center of mass C. Under action of the moment of forces $m_O = Pa \cdot \sin\theta$ the axis of whirligig z will start to turn around of point O with angular velocities $\dot{\theta}$ and $\dot{\psi}$. The vector of absolute angular velocity of whirligig will be written $\vec{\omega} = \vec{\dot{\varphi}} + \vec{\dot{\theta}} + \vec{\dot{\psi}}$. If the moment of inertia of whirligig J_z, and its velocity of rotation $\dot{\varphi}$ is much more than axis angular velocities $\dot{\theta}$ and $\dot{\psi}$, the approximate angular momentum $\vec{K}_{Oa} \approx \vec{K}_O \approx J_z \cdot \vec{\dot{\varphi}}$ will be directed along an axis z (see fig.1*a*), and the theorem of the moments (1) will be written $\dot{\vec{K}}_{Oa} \approx \vec{m}_O$. As the vector of the forces moment m_O is perpendicular to K_{Oa}, then the angular momentum will not change in its value: $K_{Oa} = J_z \cdot \dot{\varphi} = const$, and its change $\Delta\vec{K}$ in a direction will be defined by the vector m_O. Then in a plane x_1Oy_1 in time Δt the angular momentum will receive an increment $\Delta K = \dot{K}_{Oa} \cdot \Delta t = M \cdot \Delta t$ that will lead to its turn concerning an axis z_1 on an angle $\Delta\psi$. Having expressed an increment of the angular momentum through an increment of a angle: $\Delta K = \dot{K}_{Oa} \cdot \sin\theta \cdot \Delta\psi$, we receive velocity of rotation of the angular momentum: $\dot{\psi} = \lim_{\Delta t \to 0} \Delta\psi / \Delta t = m_O / (K_{Oa} \cdot \sin\theta)$. From the fig. 1a it is visible, that at turn K_{Oa}, and, hence, whirligig axis on an angle $\Delta\psi$ the moment of force m_O also will turn on the same angle concerning an axis z_1. As in new position the force action will not change, an whirligig axis will precess continuously anti-clockwise with a precession velocity $\dot{\psi}$. After substitution of the moment m_O in the precession velocity $\dot{\psi}$ it will be written $\dot{\psi} = Pa / (J_z \cdot \dot{\varphi})$.

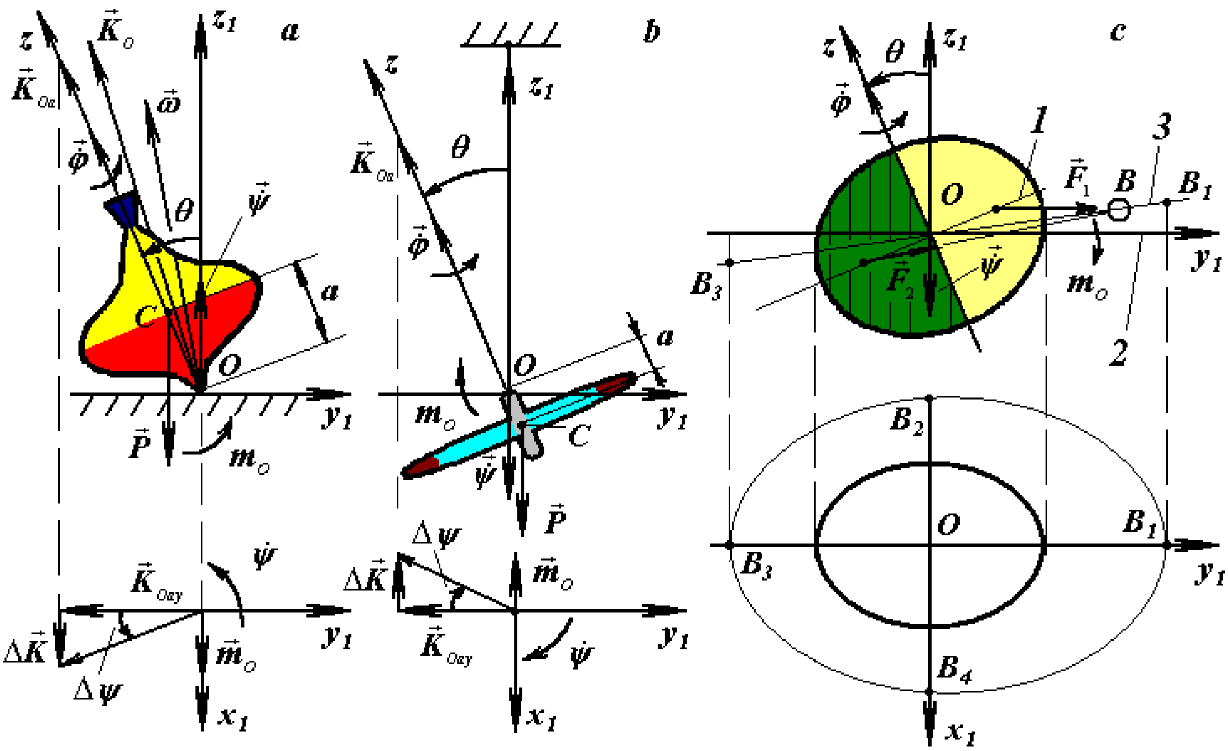

Fig. 1 Precession of the rotating bodies: *a* - whirligig on a basic surface x_1Oy_1; *b* - suspended in p. *O* wheels; *c* - the free Earth. 1 and 2 - planes of equator of the Earth and its orbit; 3 - a plane of an orbit body *B*, which act on the Earth.

As one see from the fig. 1b, in case of the hanged up wheel the moment of forces is directed clockwise. Therefore precession axes of a wheel will occur clockwise, and its velocity $\dot{\psi}$ will be defined by the same expression.

The fig. 1*c* shows the influence of body *B* on the rotating Earth. In case of the central-symmetric Earth the action of body *B* on near and far from a body of a part of the Earth will be expressed in the form of forces $\vec{F}_1$ and и $\vec{F}_2$, the resultant of which will pass through *O*. For the flattened to equator Earth, the force $\vec{F}_1$ will increase, and force $\vec{F}_2$ will decrease, therefore there will be the moment of forces m_O directed clockwise, as well as on fig. 1*b*. Therefore the axis of the Earth will precess clockwise, and precession velocity will be described by the same formula $\dot{\psi} = m_O /(K_{Oa} \cdot \sin\theta)$.

From the analysis of influence of a body to the Earth (see fig. 1*c*) the periods of precession fluctuations of the terrestrial axis follow. The Earth will be subject precession and nutation to fluctuations with half-cycles of the planets orbit time, the Sun and the Moon. Besides the axis of the Earth will test fluctuations with the periods of rapprochements of the Earth with planets in points B_1 and B_3.

3. The differential equations of rotation. The derivation of the differential equations was always spent with a sight on their approached decision. Therefore for three-century history of their creation they practically anywhere completely are not resulted. Besides their conclusion is carried out in several ways with plural transitions between systems of coordinates. As a result of their analysis we have found a more direct way with exception of intermediate transitions.

We shall determine the position of the system of coordinates rotating together with the Earth *xyz* (see fig. 1*c*) relatively axes in non-rotative system of coordinates $x_1y_1z_1$ by means of Eulerian angles: φ, ψ, θ, where ψ - a precession angle, which defines position in a plane x_1Oy_1 the lines *OK* of equator plane crossing. The moments of forces in directions of axes of the system of coordinates OKz_1z are equal the derivative of force function *U*: $M_{\dot{\psi}} = M_{z1} = \partial U / \partial \psi$, $M_{\dot{\theta}} = M_K = \partial U / \partial \theta$, $M_{\dot{\varphi}} = M_z = \partial U / \partial \varphi$. Let's design the right and left part of the theorem of the moments (1) on these axes:

$$\dot{K}_{O\dot{\psi}} = \partial U / \partial \psi, \quad \dot{K}_{O\dot{\theta}} = \partial U / \partial \theta, \quad \dot{K}_{Oz} = \partial U / \partial \varphi. \tag{2}$$

The further derivation is carried out in following sequence. The projections of angular momentum K_x, K_y, K_z on an axis of a rotating system of coordinates *xyz* are defined. Their derivatives are expressed through Euler's angles: φ, ψ and θ and with their help projections $\dot{K}_{O\dot{\psi}}, \dot{K}_{O\dot{\theta}}, \dot{K}_{Oz}$, on an axis of system of coordinates OKz_1z are defined. Force function *U* is also expressed in Euler's angles. After substitution of these variables in the theorem of the moments (2) it is received the differential equations of Earth rotation:

$$\ddot{\psi} = -2\dot{\psi}\dot{\theta}ctg\theta + \dot{\theta}\frac{J_z\omega_E}{J_x \sin\theta} - \sum_{i=1}^{n}\frac{3GM_iE_dJ_z}{r_i^5 J_x}\{0.5\sin(2\psi)(x_{1i}^2 - y_{1i}^2) - - x_{1i}y_{1i}\cdot\cos(2\psi) + z_{1i}ctg\theta(x_{1i}\cos\psi + y_{1i}\sin\psi)\}; \quad (3)$$

$$\ddot{\theta} = 0.5\dot{\psi}^2\sin(2\theta) - \frac{J_z\omega_E\dot{\psi}\sin\theta}{J_x} - \sum_{i=1}^{n}\frac{3GM\cdot E_dJ_z}{2r_i^5 J_x}\{\sin(2\theta)[x_{1i}^2\sin^2\psi + + y_{1i}^2\cos^2\psi - z_{1i}^2 - x_{1i}y_{1i}\sin(2\psi)] + 2z_{1i}(x_{1i}\sin\psi - y_{1i}\cos\psi)\cos(2\theta)\}; \quad (4)$$

$$\dot{\varphi} = \omega_E - \dot{\psi}\cdot\cos\theta . \quad (5)$$

where $\omega_E = 7.29\cdot10^{-5}$ *1/sec* a projection of absolute angular velocity of the Earth to its axis *z*.

$J_y = J_x$ - the moment of inertia of the axisymmetric Earth in a plane of equator;

$E_d = (J_z - J_x)/J_z$ - dynamic ellipticity of the Earth.

x_{1i}, y_{1i}, z_{1i} – coordinates of bodies M_i which act on the Earth.

As a result of the numerical solution of a problem of interaction of planets, the Sun and the Moon we have received changes of parameters of their orbits ranging up to 100 million years. On parameters of orbits position of an orbit and its form is defined, and position of a body in a present situation of time pays off under formulas of elliptic movement. Thus, the algorithm which allows to define coordinates of bodies influencing the Earth, at any moment from a considered interval of time is developed.

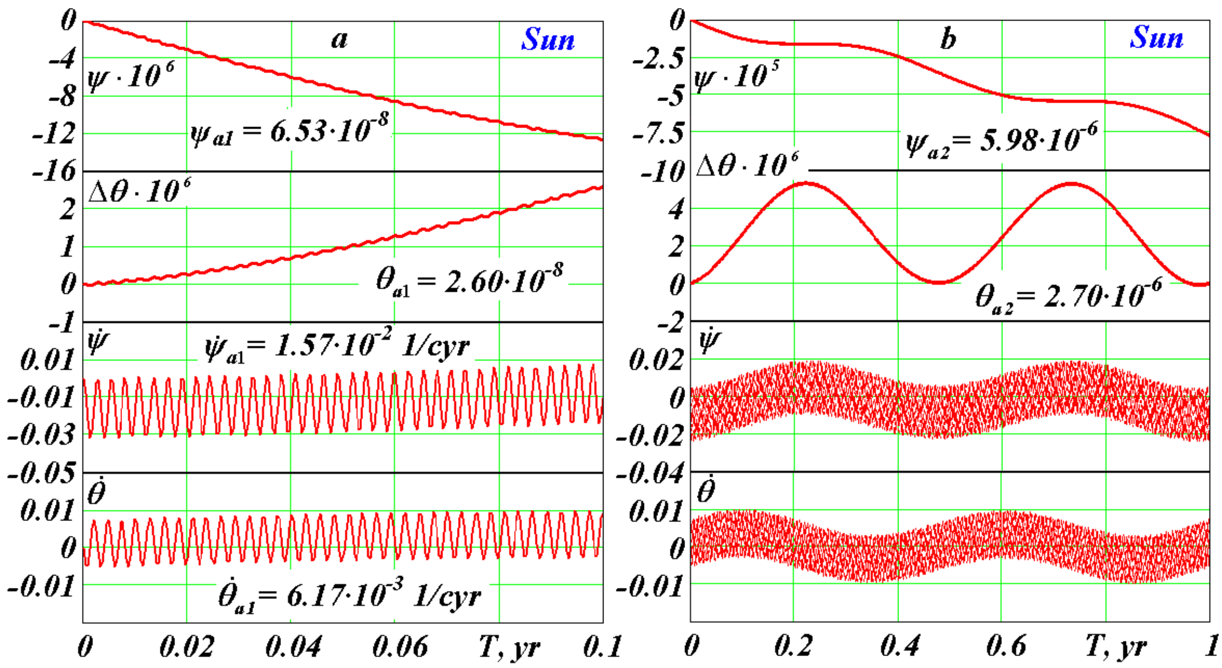

Fig. 2. Action of the Sun on rotation of the Earth: *a* - for 0.1 years, *b* - for 1 year. Precession ψ and nutation θ angles are signed in radians, and velocities $\dot{\psi}$ and $\dot{\theta}$ - in radians in century.

4. Results. In the fig. 2*a* the results of integration of the equations (3) - (4) for 0.1 years are presented at influence of the Sun on rotation of the Earth. The precession angle ψ, since zero value, decreases, making fluctuations by amplitude and period T_1 = 0.9935 days. The nutation angle θ is presented in the form of a difference $\Delta\theta = \theta - \theta_0$. From the graphics of $\Delta\theta(T)$ it is visible, that the angle of a mobile plane of equator to motionless ecliptic decreases, creating fluctuations with amplitude $\Delta\theta_{a1}$ and same period T_1. Precession rate $\dot{\psi}$ with period T_1 and amplitude $\dot{\psi}_{a1}$ oscillates around of some average value. The nutation rate oscillates with the same period T_1 and amplitude $\dot{\theta}_{a1}$ around of zero value. In the fig. 2*b* dynamics of an axis of the Earth for 1 year is presented. The precession angle increases clockwise, making fluctuations with period T_2=0.5 of year and amplitude ψ_{a2}. The precession angle oscillates with same period T_2 and amplitude θ_{a2} around of some average value.

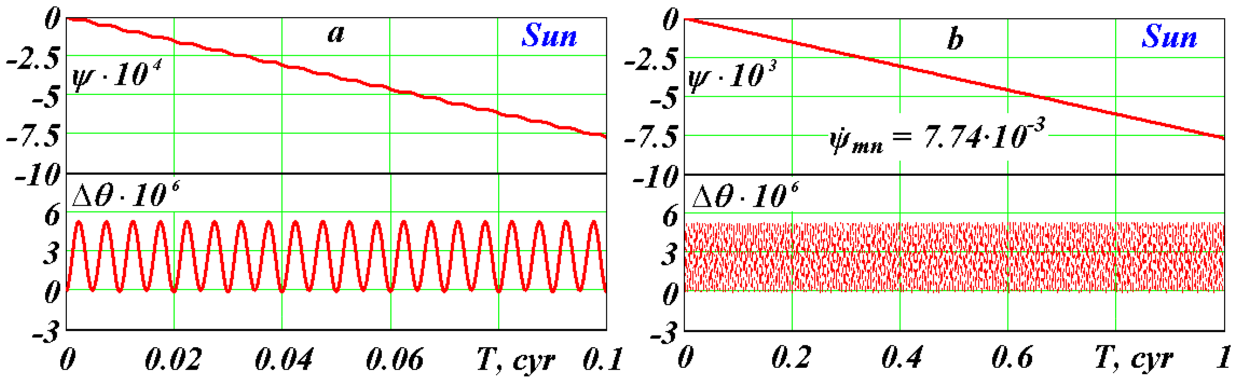

Fig. 3. Action of the Sun on rotation of the Earth: *a* - for 10 years, *b* - for 100 years. Designations see in fig. 2.

In the fig. 3*c* dynamics of an axis for 10 years is presented. The precession angle ψ with the semi-annual fluctuations, as it noted earlier, changes practically linearly and the nutation angle –with semi-annual period T_2 oscillates around of average value. Dynamics of an axis of the Earth for 100 years is shown in the fig. 3*b*. The precession angle ψ changes linearly with average velocity $\dot{\psi}_{mn} = 7.74 \cdot 10^{-3}$ rad/cyr. The nutation angle in the form of $\Delta\theta$ changes with the daily and semi-annual periods in constant limits. The received results are consistent to a coordinated with data of observation and results of other authors.