

# System Galactica: user's manual

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## 1. Essentials of Galactica

System Galactica for gravitational interaction of bodies in the form of material points was created in the beginning of 2012. Then ideas of making similar systems for other interactions had appeared. As their structure is similar to system Galactica, I have decided to make them in the form of additional modules for system Galactica. They will be added in description by separate items, and in the file system by new folders. System Galactica with Coulomb's interactions represented in section 14. Further till section 14 the manual of system Galactica with gravitational interaction between material-point bodies is followed.

The initial Galactica is a system intended for solving problems in mechanical interaction of material-point bodies. The interaction is assumed Newtonian. Galactica files are located in folder GalactcW. This folder contains other folders, InCndFls, InpFiles and RunFiles, and also files GalDiscrp.pdf and GalDiscrE.pdf that contain respectively a Russian and English description of Galactica. The zipped folders can be downloaded at <http://www.ikz.ru/~smulski/GalactcW/GalacFoldrs.zip>.

The system includes named executable files, for instance, glc3b3c, to be run on a supercomputer under Unix OS, or glc3b3c.exe, to be run under DOS; an input file galacf26.dat, and a named initial data file specifying the interacting bodies and the initial conditions of their

motion, for instance, sn69JPA2.dat. Below, the latter file will be referred to as the initial condition file.

The results yielded by an exe-file are produced as output files. The representation form is defined by keys specified in the input file galacf26.dat. Following a certain number of steps, characteristics of body motions can be issued; files containing such data are named 1, 2, 3... garez28.dat. The file garez28.dat is always produced on completion of the program. This file contains characteristics of bodies at the final calculation step. This is the main form of problem solution output.

Besides, if certain keys are set in the input file galacf26.dat, then parameters defining the trajectory of bodies whose numbers are also defined in the input file are issued, too. The names of the trajectory files are as follows: 1t, 2t, 3t ... traekt.prn. The file size is defined by the volume of the data set in the input file.

If certain keys are set, then a file dice.dat is issued. The file contains data on the time of close approach of bodies, minimal distance between bodies, and the number of a body approached by the given body. Each line in the file dice.dat is formed on accomplishment of a certain number of integration steps. The number of such steps is specified in the file galacf26.dat.

With a certain key set, a binary file (or, depending on the particular computer, an unformatted file) named galgon4 is issued. This file contains the state of the problem in binary form at the last calculation step. On subsequent start of the program, calculation is continued starting from this step, the data contained in the input file and in the initial condition file not being read out. The latter procedure is required in solving problems with long running times. The interruptions in calculations do not worsen the solution accuracy.

In the case of an emergency situation, an error file ErrGlctca is generated; this file contained a description of the error having occurred. Such errors can be encountered in opening files and their reading, and they also can be a result of insufficient array dimensions or impossibility to meet accuracy requirements in calculation of bodies merging together on collision.

In Galactica, the possibility to produce a screen of data is provided. Those parameters common to all bodies allows one to monitor both the execution of the program and the calculation accuracy. In solving a problem on a supercomputer, such data are saved in an output stream file.

In a version of Galactica with PC display graphics, along with common problem parameters, images of bodies with their velocity vectors can be produced. The form of images is defined by keys and parameters set in the input file galacf26.dat.

## 2. Differential equations of motion for material-point bodies

In modern mechanics, a lot of extraneous features have been accumulated masking the clear idea of how interactions are to be calculated. Clarifications concerning the algorithm adopted in Galactica can be found in [1-3].

All quantities in the initial condition and integration data files are used in normalized form. The normalized values results from the reduction of differential equations to non-dimensional form. Consider the equations in more detail.

According to the gravity law, a body with number  $k$  attracts a body with number  $i$ , the attractive force being

$$\vec{F}_{ik} = -G \frac{m_i m_k}{r_{ik}^3} \vec{r}_{ik}, \quad (1)$$

where  $G$  is the constant of gravitation, and  $\vec{r}_{ik}$  is the radius-vector that connects the massive body  $m_k$  with the massive body  $m_i$ .

If the total number of bodies is  $n$ , then the  $i$ -th body will be acted upon by all other bodies, with the net interaction force being

$$\vec{F}_i = -Gm_i \sum_{k \neq i}^n \frac{m_k \vec{r}_{ik}}{r_{ik}^3}. \quad (2)$$

The second law of mechanics,  $\vec{a} = \vec{F} / m$ , states that, under the action of force (2), the  $i$ -th body will set in motion relative to an inertial (non-accelerated) reference system with acceleration

$$\frac{d^2 \vec{r}_i}{dt^2} = -G \sum_{k \neq i}^n \frac{m_k \vec{r}_{ik}}{r_{ik}^3}, \quad i = 1, 2, \dots, n, \quad (3)$$

where  $\vec{r}_i$  is the radius-vector of the body  $m_i$  relative to some center in the inertial frame of reference. For instance, in treating the interactions between solar-system objects  $\vec{r}_i$  is the radius-vector of the body  $m_i$  with respect to the center of mass of the Solar system.

Equation (3) gives a system of  $3n$  nonlinear second-order differential equations,  $n$  being an arbitrary number. The total number of bodies  $n$  is specified in the input file galacf26.dat. Besides, the arrays in the program should have dimensions not less than  $n$ . For instance, the ex-file glc3b3c enables calculation of problems involving up to  $n = 300$  bodies.

Differential equation (3) is written in a non-accelerated (inertial) frame. However, all bodies move with acceleration, and only their center of mass  $C$  in the system of interacting bodies is non-accelerated (on the condition that the action of other bodies on the system of interest can be neglected). As a rule, observations are being made to measure the coordinates and velocities relative to some other body used as a frame. For instance, in the Solar system the coordinates of a massive body  $m_i$  can be reckoned from the Sun (S); such coordinates are denoted as  $x_{Si}$ ,  $y_{Si}$ ,  $z_{Si}$ , and velocities,  $v_{xSi}$ ,  $v_{ySi}$ ,  $v_{zSi}$ . The latter coordinates and velocities can be used to calculate the coordinates and velocities of the center of mass of the Solar system, which, in projection onto the axis  $x$ , have the form

$$X_C = \sum_{i=1}^n m_i \cdot x_{Si} / M_{Ss}; \quad V_{xC} = \sum_{i=1}^n m_i \cdot v_{xSi} / M_{Ss}. \quad (4)$$

Here,  $n$  is the total number of bodies in the Solar system, and  $M_{Ss} = \sum_{i=1}^n m_i$  is the mass of the Solar system or, generally, the mass of all interacting bodies.

Note that the motion of a selected body, say, the Sun, varies in time, whereas the reference frame keeps moving without acceleration. That is why the frame relates to a certain time in the motion of the chosen body. For instance, the frame attached to the Earth's equatorial plane refers to the beginning of a 50-year period: system of 1900.0, system of 1950.0, or system of 2000.0.

The coordinates and velocities of bodies relative to the center of mass  $C$  taken in projection onto the axis  $x$  are

$$x_{Ci} = x_{Si} - X_C; \quad v_{xCi} = v_{xSi} - V_{xC}. \quad (5)$$

Here, the coordinates are expressed in meters; time, in seconds; and masses, in kilograms.

With variables normalized by a characteristic size of the system of interacting bodies  $Am$  and their mass  $M_{Ss}$ , equations (3) acquire non-dimensional form [1-2]. For the Solar system, we define the quantity  $Am$  so that to render the non-dimensional time  $T$  expressed in sidereal centuries. Here, the quantity  $Am$  turns out to be close to the semi-axis of the Pluto orbit. In non-dimensional form, the equation projected onto the axis  $x$  is

$$\frac{dv_{xi}}{dT} = - \sum_{k \neq i}^n \frac{m_{ok} (x_i - x_k)}{r_{ik}^3}. \quad (6)$$

Here,  $x_i = x_{Ci} / Am$  is the non-dimensional coordinate of the  $i$ -th body;  $m_{ok} = m_k / M_{Ss}$  is the non-dimensional mass of the  $k$ -th body;  $r_{ik} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2}$  is the non-dimensional distance between the  $i$ -th and  $k$ -th body;  $v_{xi} = v_{xCi} \cdot k_v$  is the non-dimensional velocity

of the  $i$ -th body;  $k_v = \sqrt{\frac{Am}{G \cdot M_{Ss}}}$  is the velocity coefficient in sec/m;  $T = t \cdot k_t$  is the non-dimensional time; and  $k_t = \sqrt{\frac{G \cdot M_{Ss}}{Am^3}}$  is the time coefficient in sidereal centuries per sec.

In initial condition files, in solar-system problems the non-dimensional time  $T = t \cdot k_t$  is expressed in sidereal centuries, each century comprising  $P_{sdc} = 36525.636042$  days.

All quantities in Galactica are used as non-dimensional quantities, including the radii of bodies ( $Ra_o$ ), their momenta ( $P_{ox}$ ), angular momenta ( $M_{ox}$ ), kinetic ( $E_o$ ) and thermal ( $E_{to}$ ) energies. Given two scales  $Am$  and  $M_{Ss}$ , the coefficients  $k_v$  and  $k_t$  can be found. Afterwards, the solution can be rendered dimensional using the formulas

$$\begin{aligned} t = T/k_t; \quad x_{mi} = x_i \cdot Am; \quad v_{mi} = v_i/k_v; \quad Ra_i = Ra_{oi} \cdot Am; \quad P_x = P_{ox} \cdot M_{Ss}/k_v; \\ M_x = M_{ox} \cdot M_{Ss} \cdot Am/k_v; \quad E = E_o \cdot M_{Ss}/k_v^2; \quad E_t = E_{to} \cdot M_{Ss}/k_v^2. \end{aligned} \quad (7)$$

If the primary scale is the time  $T$  expressed, say, in sidereal centuries  $P_{sdc}$ , then the scale length  $Am$  entering the expression for  $k_t$  can be calculated as

$$Am = (G \cdot M_{Ss}/k_t^2)^{1/3}, \quad (8)$$

where  $k_t = 1/(P_{sdc} \cdot 24 \cdot 3600)$ .

In mode  $K11 = 0$ , i.e. in operation with input data and initial conditions specified for a distributed region filled with substance, a density of bodies  $\rho_b$  expressed in  $\text{kg/m}^3$  is used. Given the scale  $Am$ , from non-dimensional radius  $Ra_{oi}$  and mass  $m_{oi}$  of bodies one can calculate dimensional quantities:

$$Ra_i = Ra_o \cdot Am; \quad m_i = m_{oi} \cdot 3 \cdot \pi \cdot Ra_i^3 \cdot \rho_b / 4. \quad (9)$$

Then, the total mass of bodies  $M_{Ss} = \sum_{i=1}^n m_i$  and the coefficients  $k_v$  and  $k_t$  can be calculated.

Further calculations are to be carried out by formulas (7).

Thus, the solution produced by Galactica can be recalculated, using scaling factors  $Am$  and  $M_{Ss}$ , to various situations. For instance, the problem on evolution of substance ending in the formation of a star and planets can be employed for stars with a widely ranging distribution of substance. The data obtained in solving this problem can also be recalculated to scales over which galaxies form. On the contrary, with values  $Am$  and  $M_{Ss}$  reduced to a micro-level one can employ the results for calculating the interaction of bodies at micro-level.

### 3. Solution method

In Galactica, differential equations of motion (6) are integrated as follows. The value of a function at a next time  $T=T_0 + \Delta T$  is calculated using a Taylor series; this series, for instance, for the coordinate  $x$  is

$$x = x_0 + \sum_{k=1}^K \frac{1}{k!} x_0^{(k)} (\Delta T)^k, \quad (10)$$

where  $x_0^{(k)}$  is the derivative of order  $k$  at the time  $T_0$ .

The value of velocity  $x' = v_x$  is calculated by a similar formula, and the acceleration  $x_0'' = \frac{dv_{xi}}{dT}$ , by Eqs. (6). Higher derivatives  $x_0^{(k)}$  are calculated using analytical expressions that we had derived by differentiating Eqs. (6). Presently, a six-order calculation scheme is used in Galactica, i.e.  $K=6$ . Here, the sixth derivative is calculated from the difference of the fifth derivatives. Addition of each derivative, except for the sixth one, improves the solution accuracy by three orders. The sixth derivative improves the accuracy by 2.5 orders. This method ensures a high accuracy and a good solution dynamics. For instance, in double-length calculations (17 decimal digits) the inaccuracy does not grow in value as calculations are continued.

In adjusting the integration procedure, more than ten various methods for controlling accuracy were tested [4, 5]. Some of those methods are used in Galactica for monitoring the inaccuracies. Data concerning the accuracy are issued in output files. Some relevant data were

reported in [4, 5] and in other publications; such data will also be discussed below in other sections. From problem solution data, the user can judge the accuracy and, if necessary, take measures to improve it.

In the present manual, the Galactica algorithm is not described, some elements, however, will be outlined below as they soon as they become necessary for discussion of integration data.

#### 4. Input file galacf26.dat

##### 4.1. General description

Table 1 lists enumerated parameters involved in the input file galacf26.dat. The numbers of parameters are given in column 1 of the table. All parameters can be subdivided into four groups: 1) interaction region parameters; 2) general problem solution parameters; 3) parameters regulating the output of the trajectories of separate bodies; 4) parameters defining on-screen data display.

The digits with point in the fourth column of Table 1 are real quantities, and the digits without point are integer numbers. The real values are represented in format F11.4; the integer numbers Klmax, Ltk, Ltk2, k2, Kl3, Kl11, and Kli, in format I12; and others, in format I5. The string variable is used in format A12.

In the second column of Table 1, first row, the date of creation of the file version is additionally given, for instance, 01.02.2012.

##### 4.2. Interaction region parameters

The first group, lines 1÷20 and 22 in the table, comprises parameters for setting input data and initial conditions of interacting bodies resulting from a uniform distribution of substance in a certain space region. The region is defined in lines 1-3 of Table 1 as a parallelepiped with sides A, B, and C. The side lengths are relative values, normalized by the largest length A, so that  $A = 1$ . The origin has coordinates X0, Y0, and Z0. Along the axis X, the region is partitioned into Mu intervals. The lengths B and C are also divided into intervals, identical in length and close in value to the partitioning intervals for the side A. The mass of the whole region is defined by the volume  $A \cdot B \cdot C$  and absolute density Roa.

Table 1. Contents of the input file

No	Parameter	Designation	Digit starting from the 60 <sup>th</sup> position
1	Length of region along axis X ( $A = 1$ ) 03.01.2016	A	1.
2	Height of region along axis Y ( $B \leq 1$ )	B	1.
3	Thickness of region along axis Z ( $C \leq 1$ )	C	1.
4	Initial X coordinate of region	X0	0
5	Initial Y coordinate of region	Y0	0
6	Initial Z coordinate of region	Z0	0
7	Number of divisions over axis X	Mu	2
8	Relative initial density of region	Ro0	1.
9	Relative initial density of point body	RoS	2.
10	X coordinate of a separate body	Xs	1.5
11	Y coordinate of a separate body	Ys	0.5
12	Z coordinate of a separate body	Zs	0.5
13	Velocity of the separate body along axis X	Us	-0.4
14	Velocity of the separate body along axis Y	Vs	-0.15
15	Velocity of the separate body along axis Z	Ws	0.5
16	Angular velocity of rotation of region	wu	0.0
17	Relative X coordinate of the axis of rotation	X01	0.5
18	Relative Y coordinate of the axis of rotation	Y01	0.5
19	Absolute average density of region, kg/m <sup>3</sup>	Roa	1.E-7
20	Absolute density of bodies, kg/m <sup>3</sup>	Rob	5000.
21	Depiction with respect to the first body (see below)	Kl10	0
22	Integration time step at Kl1=0	dt	5.0e-4

23	Maximum dimension of TR array	Klmax	6000
24	Number of integration steps in one cycle	Ltk	1000
25	Number of cycles	Ltk2	1
26	Key: input data from file 4	K11	2
27	Key: start of counting time/Step correction	K14	1
28	Search of close approaches for body No. at K13	Nbd	0
29	Key: writing data to file	K17	0
30	Number of interacting bodies	k2	12
31	Interval of data (picture) output (in steps)	K13	500
32	Beginning of step-by-step data (picture) output	K111	0
33	Key: writing trajectory parameters	K112	0
34	Interval of trajectory parameter output (in steps)	Kli	50
35	Display mode	Nre	0
36	Correction of area size on the screen	K114	1
37	Coefficient of area form on the screen	af	0.72
38	Coefficient of mass size of the screen	Rd	0.08
39	Coefficient of velocity on the screen	amv	3.
40	Correction factor of step dtk at bodies' approach	bt	0.5
41	Flat depiction of layer to Z	Ke	0
42	Rotation of area relative to X axis through angle	al	10.
43	Rotation of area relative to Y axis through angle	bet	20.
44	Beginning of screen area along x	Xe0	300.
45	Beginning of screen area along y	Ye0	280.
46	Image size in x	aix	150.
47	Image size in y	biy	70.
48	Initial data file name	FIMA4	sn2010Ap.dat
49	Number of bodies in trajectory analysis	Np	2
50	Key: body velocity output	K115	0
51	Velocity color of the first body	ipp(1)	3
52	Velocity colour of the all bodies	jpp(1)	4
54	Number of the first body	kpp(1)	1
55	Velocity color of the second body	ipp(2)	5
56	Free key of the second body	jpp(2)	1
57	Number of the second body	kpp(2)	2
58	Velocity color of the third body	ipp(3)	8
59	Free key of the third body	jpp(3)	1
60	Number of the third body	kpp(3)	4

Apart from bodies obtained by partitioning the region into identical volumes, a separate massive body can be specified. Its mass is defined by the relative density RoS (line 9). The value of RoS is normalized by the relative density of the region Ro0, so that  $Ro0 = 1.0$ . The parameters Xs, Ys, and Zs define the relative coordinates of the separate body. If the separate body is located at a distance exceeding the distance between the existing bodies, then this body is added to the total number of bodies.

The radii and masses of bodies are calculated from their volumes and absolute density Rob assuming the bodies spherical. The coordinates of bodies are specified at nodes, the velocities being set zero. The velocity of the separate body is defined by parameters Us, Vs, and Ws. The masses of all bodies are normalized by the mass of the whole system.

In specifying the initial velocities of bodies, a possibility to set rotation of the whole region at an angular velocity wu is provided. The position of the axis of rotation is defined by the coordinates X01 and Y01. The axis of rotation is parallel to the Z-axis.

For integration of such problems, the integration step is defined in line 22 as the parameter dt. The whole system of initial conditions for bodies is to be generated in the center-of-mass system. However, with an additional separate body introduced, the system of bodies no

longer remains attached to the center of mass. As a result, due to the interaction in the system, a directional motion may emerge.

#### 4.3. General problem solution parameters

This group of parameters is contained in lines 21, 23÷32, 40, and 48 of Table 1.

Let us begin with line 24; line 23 will be considered below. The quantity  $Ltk$  is the total number of integration steps. This number should be smaller than the maximum value of double-precision integer numbers, 2147483647. If necessary, a greater amount of steps can be set in parameter  $Ltk2 > 1$ . This parameter defines the number of cycles to be performed repeatedly, each cycle comprising  $Ltk$  steps.

Key  $K11$  takes five values: 0, 1, 2, 3, or 4. With  $K11 = 0$ , the initial condition file is not used, and all problem data are defined by parameters described in Sec. 4.2. With  $K11 = 1$ , input data and initial conditions are read out from the output file `garez28.dat`. Here, previous problem solution is being continued. However, the accuracy will decrease due to the round-off on passing over from binary to decimal digits at the moment of generation of file `garez28.dat`. In most cases, such a worsening of accuracy is insignificant. Prior to starting the program, modification can be applied to the step  $dT$  in the file `garez28.dat`. This allows certain calculations to be performed with regulated accuracy.

With  $K11 = 2$ , input data and initial conditions are read out from the file whose name is specified in line 48 as parameter `FIMA4`. Next two values 3 and 4 of the key  $K11$  turn on the calculation continuation mode with unchanged accuracy. Here, after each calculation stage, defined by keys  $K17$  and  $K13$ , an unformatted (binary) file `galgon4` is generated. On a new start of the program, reading data out of the input file and initial condition file is ignored; instead, all conditions are read out from the file `galgon4`. If necessary, modification can be applied to some conditions. For that, an additional code, not a component of Galactica, was written. The values of the keys differ in that during the first run of the program with  $K11 = 3$  solution starts with the file `garez28.dat`, whereas with  $K11 = 4$ , with the file specified by parameter `FIMA4`.

In the first run with keys  $K11 = 2$  or 4 the first time step is reduced by one hundred time, i.e. it is set equal to  $0.001 \cdot dT$ .

Key  $K14$  in line 27 controls two functions: the start of the count of time and the correction of step  $dT$ . If the key  $K14 = 1$  or  $K14 = 3$ , the count of time  $T$  starts from the value given in the initial condition file, whereas with  $K14 = 0$  or  $K14 = 2$  the time count starts from  $T = 0$ . If key  $K14 = 2$  or  $K14 = 3$ , the mode of the step  $dT$  correction is switched. The step is doubled if the average change in the angular momentum difference  $\Delta M_z$  for the last five steps is increased by 10 percent.

Parameter  $Nbd$  specifies the number of the body for which close approaches to minimal distances with other bodies will be recorded in memory over the step interval  $K13$ . In the latter case, a file `dice.dat` is to be generated, with registering close-approach characteristics after  $K13$  steps; those characteristics include the time, the minimum distance between the bodies, and the number of the approached body. With  $Nbd = 0$ , close approaches are not registered, no file `dice.dat` being generated.

With  $K17 = 1$  in line 29 after  $K13$  steps integration data over intermediate intervals are issued. Those data are accumulated in files 1, 2, 3 ... `garez28.dat`. The total number of generated files is defined as  $Ltk \cdot Ltk2 / K13$ . When starting a program, one should observe that the number of such files would not appear too large. With  $K17 = 0$ , no intermediate data outputs are produced.

Parameter  $k2$  in line 30 defines the number of interacting bodies. If arrays in the executable program has a dimension smaller than  $k2$ , then a message «\*08G Change the dimension of arrays \*\*\* Change l2 >= k2» will be output to the display screen and also to the error file `ErrGlctca`, where  $k2$  is a set number. In the latter situation, one has to use a program treating a greater number of bodies. It also should be kept in mind that from the initial condition file parameters for  $k2$  bodies will be read out no matter for how many bodies parameters are available in the file.

In line 31, parameter K13 defines the number of steps following which integration data are output to files: 1, 2, 3 ..., and also to the PC display (or to supercomputer's output stream file). It should be borne in mind that too frequent data output makes the counting rate of Galactica seriously deteriorated.

Key K111 in line 32 sets the number of the step on which on during ten steps  $dT$  step-by-step data output will be produced. This key makes it possible to thoroughly examine the situation encountered following a long time from the start of calculations. With  $K111 = 0$ , this option is made inactivated.

In line 40, the correction coefficient  $bt$  for step  $dT$  for approaching bodies is specified. The value of  $bt$  can be varied if, on an encounter of bodies, the calculation of their merge becomes difficult.

In line 48, parameter FIMA4 defines the name of initial data and condition file. The file name can be specified as a string variable involving maximum of twelve symbols.

#### **4.4. Parameters for output of trajectories of selected bodies**

For investigation of the trajectory of a selected body one has to know the kinematic characteristics of the body with sufficiently high time resolution. For that purpose, parameters 33-34 and 49-60 are intended. Key  $K112 = 1$  in line 33 turns on the option of writing trajectory parameters in a file. Here, kinematic data of certain bodies are to be chosen following a given number of steps and after accumulating their amount defined in parameter  $Klmax$ , such data are output to trajectory files 1t, 2t, 3t ... traekt.prn. Parameter  $Kli$  defines the number of time steps  $dT$  following which kinematic characteristics of bodies are sampled.

Parameter  $Np$  in line 49 defines the total number of the bodies for which trajectory characteristics will be issued. Key  $K115 = 1$  in line 50 turns on the option of issuing the velocity components. With  $K115 = 0$ , only the coordinates of the bodies are issued.

In subsequent lines, parameter threes  $ipp()$ ,  $jpp()$ ,  $kpp()$  set a particular body. The number of the body is set by the parameter  $kpp()$ . Parameter  $ipp()$  sets the color of the velocity vector in using the Galactica version operating in graphic mode. Parameter  $jpp(1)$  sets the color of the velocity vector of other bodies. Other parameters  $jpp()$  is not used in the present version of Galactica. The digits in round brackets at the above parameters give the number of body sequences in trajectory files 1t, 2t, 3t ... traekt.prn. The total number of such files is defined by parameter  $Klmax$ . In the presented compilations of codes the value of  $Klmax$  should not exceed 15000. In the same codes the total number of bodies  $Np$  should not exceed three for  $K115 = 1$ , and it should not exceed six at  $K115 = 0$ . For setting additional three bodies, to file galacf26.dat three lines with parameters  $ipp()$ ,  $jpp()$  and  $kpp()$  for each body are to be added.

In case of erroneous setting of the above parameters, a message «\*05G Change the dimension of TR \*\*\*  $K112d0 = 750$ ,  $K112e0 = 20$ » is output into the error file ErrGlctca and to the display screen. Here, 750 and 20 are set dimensions of the two-dimensional array for trajectory parameters  $TR(K112d0, K112e0)$ , where  $K112d0$  is the total number of the records, and  $K112e0$  is the number of numerical values registered in one record.

#### **4.5. On-screen display parameters**

Lines 21, 35-39, and 41-47 in Table 1 specify parameters for display of images of bodies and their velocity vectors on the screen (see Fig. 1). Parameter  $jpp(1)$  sets the color of the velocity vectors. Those parameters are used by programs operating with graphics.

Here, the coordinate system employed in the initial condition file, i.e. the system attached to the center of mass of the whole system, is used. If one wants to represent the motion of a body relative to some other body, then in line 21 the key  $K110$  is to be set to 1. In the latter case, the coordinate images and the velocity of all bodies will be issued relative to the body whose number is set with parameter  $kpp(1)$  in line 54.

As it was mentioned above, the range of steps in giving out images is specified with key  $K13$  in line 31. Parameter  $Nre$  in line 35 is used to account for the display mode. In the standard graphics Fortran the mode of display are given in Appendix 2. The influence of mode depends



on the specific computer and executable code. The required value can be selected from submitted ones in Appendix 2. The value of  $Nre = -3$  is probably the most acceptable.

Parameter  $K114$  in a line 36 is used for maintenance of visibility of bodies at unlimited expansion of system.

Parameter  $af$  in line 37 allows preservation of image shape; i.e., with it, a square is represented as a square, not a rectangle. In representation of a body and its velocity on the screen the sizes on the screen are determined as values normalized to maximum values of mass and velocity. The latter allows one to observe the bodies on variation of their masses and velocities in a broad range during problem solution. For initial representation of a body in variously configured displays correcting coefficients  $Rd$  and  $amv$  in lines 38 and 39 are used.

If the body system is represented with a multilayered region over the axis  $Z$ , then parameter  $Ke$  can be used to specify the number of the layers whose bodies will be displayed on the screen. With  $Ke = 0$ , this option is off.

In lines 42 and 43, rotations of the body system in displaying images relative to axes  $X$  and  $Y$  can be set using respectively parameters  $al$  and  $bet$ . Both angles are to be expressed in degrees.

In lines 44 and 45, parameters  $Xe0$  and  $Ye0$  specify the top left origin of the region to be displayed on the screen. The image scales over the axes  $X$  and  $Y$  can be altered using parameters  $aix$  and  $biy$ , respectively. To enlarge the image, the parameters  $aix$  and  $biy$  are to be increased in value.

As it was noted above, the velocity vectors of individual bodies can be highlighted with certain color. The number of the body to be highlighted is set by parameters  $kpp()$  in lines 54, 57, etc., and the number of the color, by parameters  $ipp()$  in lines 51, 55, etc. In standard Fortran graphic operators, the correspondence between the color and its number is defined in file  $FrtnColr.bmp$  contained in folder  $InpFiles$ .

View the image depends on the parameter  $Mu$ . It is therefore desirable to set the value of this parameter, as well as the parameters  $B$  and  $C$  in lines 2 and 3 they are in the file of initial conditions. The positions of these parameters are described in section 5.

The form of representation of integration data depends on many factors, including the Fortran and compiler version, and also the screen type and screen resolution. Given in folder  $InCndFls$  together with reference initial condition files for solved problems are images of bodies contained in  $bmp$ -files. The names of those files coincide with the initial condition files. For instance, for the initial condition file  $sn49jplc.dat$  the images are contained in the file named  $Sn49jplc.bmp$ .

Body images are also available in the folder  $InpFiles\Area$ . One can find here six examples of the input file  $galacf26.dat$ ; in the file names, the extension  $dat$  is replaced with other symbols. Those symbols denote the type of problems in which the interaction between bodies formed by a region filled with substance is considered. In such problems, body images are contained in  $bmp$ -files, where the body names are represented with symbols forming the input file name. For instance, the input file  $galacf26.64r$  represents interaction of 64 radially moving bodies, the image being given in file  $64r.bmp$ .

The input files contained in this folder give various examples of using parameters for data display; the outcomes of a particular choice of parameters are easy to trace considering graphic files.

### **5. Initial condition and integration data files**

Initial condition files, for instance, files  $sn49f.dat$ ,  $sn49jplc.dat$ , etc. in folder  $InCndFls$ , and also output files: 1, 2, 3, etc., and file  $garez28.dat$  are identical in structure. In describing the files, we will use designations of two kinds: 1) unsubscripted Roman type, and 2) italic type. The latter designations are used in programs, and the former designations, in printed materials.

Each of the above-mentioned files begins with a set of 24 numbers:  $T0$ ,  $omm$ ,  $Um$ ,  $dTp$ ,  $Px$ ,  $Py$ ,  $Pz$ ,  $AMx$ ,  $AMy$ ,  $AMz$ ,  $Spsx$ ,  $Spsy$ ,  $Spsz$ ,  $E$ ,  $Em$ ,  $Ett$ ,  $dT$ ,  $i2b$ ,  $j2b$ ,  $k2b$ ,  $A$ ,  $B$ ,  $C$ , and  $Mu$ . Those numbers are common to all interacting bodies. Then, fifteen numbers for each  $k$ -th body

follow: om(k), (X(k,q),q=1,3), (U(k,q),q=1,3), (dUp(k,q),q=1,3), (Sp(k,q),q=1,3), Ra(k), Et(k). Here, q is the subscript listing the variables over the three coordinates x, y, and z.

The general parameters are the following:

T0 – time in relative units, for instance, in solar-system files in sidereal centuries, each century comprising 36525.636042 days;

omm – maximal mass of body reduced by the total mass of all bodies;

Um – maximal velocity in relative units;

dTp – time step in relative units at the previous step;

Px, Py, Pz – projections of the total momentum of the whole body system on the coordinate axes;

AMx, AMy, AMz – projections of the total angular momentum of the whole body system;

Spsx, Spsy, Spsz – projections of the sum of the angular momenta due to self-rotation of bodies around their axes (spin projections);

E – total kinetic energy of all bodies at the given moment T0;

Em – maximal kinetic energy of all bodies starting from the beginning of integration;

Ett – thermal energy of all bodies which the bodies have acquired during their merging on collision;

dT – current time step in relative units;

i2b, j2b, k2b – number of nodes obtained on partitioning of the substance array over the axes x, y, and z, respectively;

A, B, C – dimensions of the array over the axes x, y, and z;

Mu – total number of divisions of the array over the axis x.

The last seven parameters are used in setting interacting bodies in the form of a substance uniformly distributed in the space with density Roa.

Next, 15 parameters for each body follow:

om(k) ÷  $m_{ok}$  – non-dimensional (relative) mass of the  $k = i$ -th body ( $k$  and  $i$  are the numbers of bodies in different texts);

(X(k,q),q=1,3) ÷  $x_i, y_i, z_i$  – non-dimensional barycentric coordinates of the  $k = i$ -th body; for solar-system files in equatorial coordinate system;

(U(k,q),q=1,3) ÷  $v_{xi}, v_{yi}, v_{zi}$  – non-dimensional velocities of the  $k = i$ -th body;

(dUp(k,q),q=1,3) ÷  $x_i^{(5)}, y_i^{(5)}, z_i^{(5)}$  – non-dimensional derivatives of the fifth order for the  $k = i$ -th body;

(Sp(k,q),q=1,3) ÷  $S_{pxi}, S_{pyi}, S_{pzi}$  – non-dimensional angular momenta (spins) due to the self-rotation of the  $k = i$ -th body. A body is assumed to get a spin if the body has formed as a result of the merge of other bodies having approached each other to a distance smaller than the sum of their radii. Initially, the rotation of bodies can be ignored, with their spins being defined as  $S_{pxi}=S_{pyi}=S_{pzi}=0$ . This strategy is adopted in the initial condition files given below.

Ra(k) – non-dimensional equivalent radius of the  $k = i$ -th body; this radius is calculated from the mass and average density of the body, the body assumed spherical;

Et(k) –  $E_{ti}$  – non-dimensional thermal energy of the  $k = i$ -th body. This energy has arisen from merging of several bodies in one body due to transformation of kinetic energy into thermal energy.

Initially, the thermal energy of bodies can be neglected, i.e. it can be set to zero,  $E_{ti}=0$ , this being the case in the provided initial condition files. As two massive bodies  $m_{oi}$  and  $m_{ok}$  merge together, the thermal energy of the resultant body is calculated as

$$E_{ti} = \frac{m_{ok}m_{oi}v_{rki}^2}{2(m_{ok} + m_{oi})}, \quad (11)$$

where  $m_{oi} \geq m_{ok}$ , and  $v_{rki}$  is the non-dimensional radial velocity of the massive body  $m_{ok}$  relative to the body  $m_{oi}$ .

On merging, the mass of the  $i$ -th body increases, assuming the value  $m_{oi} + m_{ok}$ , the mass of the  $k$ -th body being put equal to zero. The changes of coordinates, velocity and spin

projections, and body radius due to the merge of bodies are calculated using the algorithm described in Sec. 11.3 of [1, 2]).

In the last line of the initial condition file, five numbers can be written: date of generation of initial conditions, total number of bodies, mass of the system, scale size in meters, and velocity coefficient: date,  $k_2$ ,  $M_{S_0}$ ,  $A_m$ ,  $k_v$ . The date is represented as an integer number involving eight digits: year, month, day. This line is not read by Galactica, and it is absent from the calculated data file.

In Appendix 1 there is the sample of program in MathCad software for generation of initial condition file.

### 6. Trajectory files 1t, 2t... traekt.prn

As mentioned in Sec. 4.4, the names of trajectory files look as 1t, 2t, 3t ... traekt.prn. All the names look similarly except for the last file, traekt.prn. Each record refers to one time; it contains K112e numbers. The total amount of the numbers is defined by the expression

$$K112e = 3 \cdot (K115 + 1) \cdot N_p + 2, \quad (12)$$

where K115 and  $N_p$  are parameters specified in the input file galacf26.dat and described in Sec. 4.4.

The record starts from moment of time T in non-dimensional units. Then, the kinetic energy of the whole system E follows. Then, for each body, three coordinates, x, y, and z are issued. With K115 = 1, also issued are the velocities vx, vy, vz. All those parameters, including time, are represented as non-dimensional quantities (see Sec. 2).

The total number of the records in the files and their volume are defined by the parameters described in Sec. 4.4. If the total amount of records is small, then just one trajectory file traekt.prn will be generated. To the end of the file, a record comprising ten general problem parameters is appended: T, Tem, Em, Ett, AMz1, Nb, Lsls, Um, omm, and Amz,

Here, T is the time at which the calculation of the problem was over;

Tem is the time at which the kinetic energy of the system was maximal;

Em is the maximum value of the kinetic energy of the system;

Ett is the thermal energy of the system at the end of the calculation;

AMz1 is the angular momentum of the whole system relative to the axis z at the first calculation step;

Nb is the total number of bodies in the system at the beginning of the calculation;

Lsls is the number of merges, i.e. the number of bodies that, on approaching one another, formed a single body;

Um is the highest velocity in the calculation run;

omm is the mass of the largest body;

AMz is the angular momentum of the whole system relative to the axis z at the end of calculation.

If preceding files, e.g. a file 1t, are output, then the file traekt.prn has the same volume, and after the final record this file contains records left from the preceding file.

In Galactica, the kinetic energy of the system is calculated as

$$E = \sum_{i=1}^n 0.5 m_i v_i^2. \quad (13)$$

As it was noted above, on merging of two bodies the thermal energy is calculated by formula (11). As it will be shown below in Sec. 8 the angular momentum  $AM_z = M_z$  is calculated by formula (14).

### 7. Close-approach file dice.dat

As it was mentioned in Sec. 4.3, if the parameter Nbd is non-zero, then the distances to which the body with number Nbd approaches other bodies are calculated. On accomplishment of K13 calculation steps three numbers are to be written in the file dice.dat: time at which a closest approach to a certain body occurs, the number of this body, and the distance to which the bodies approach one another. The total number of such records in the file is  $Ltk \cdot Ltk_2 / K13$ .

We used this option while investigating into the approach of asteroids to planets and Moon [6-7]. Fig. 1 and Fig. 7 in [7] show data obtained on the basis of file dice.dat.

## 8. On-screen display

In solving a problem on a PC with a Galactica program with graphic option, numerical data concerning the integration process and a picture illustrating the position of planets with their velocity vectors are given out to the display screen following a certain number of integration steps (see Fig. 1). The data allow one to monitor the solution process and judge on solution reliability. In using a no-graphics version of Galactica, only numerical data are output to the screen. In solving the problem on a supercomputer, the data are issued to the output stream file.

The first 16 parameters reproduce the values of common parameters:

TO, omm, Um, dTp, Px, Py, Pz, AMx, AMy, AMz, Spsx, Spsy, Spsz, E, Em, Ett, used in the initial condition and integrated data files described in Sec. 5. The remaining three parameters are the integration step number  $L_t$ , the value of this step  $dT$ , and the change of the angular momentum  $\delta M_z$  taken in projection onto the axis  $z$ . It should be remembered here that the total number of the integration step at  $Ltk2 > 1$  also includes the number of cycles performed with a number of steps Ltk.

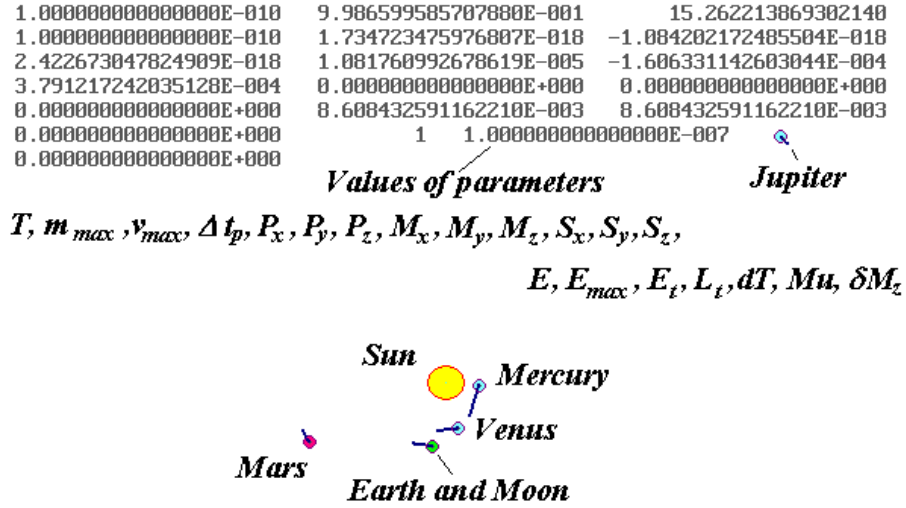


Fig. 1. On-screen representation of integration data for equation system (6) as viewed from the South Pole. Explanation and denotation of numerical parameters are given in italics.

Those quantities enable an analysis of the evolution of the system and the integration process of Eqs. (6). Each of the quantities, the largest mass  $omm = m_{max}$ , the momentum  $P$ , the angular momentum  $M$ , the sum of intrinsic angular momenta  $S$ , the kinetic energy  $E$ , the thermal energy  $E_t$  of all bodies, and the relative change ( $\delta M_z$ ) of the projection of the angular momentum  $M$  onto the axis  $z$ , allow the user to get an idea how the solution evolves and, in case of an error, to spot the error.

An important control factor is the relative change of  $\delta M_z$ . In the absence of external actions, we have:

$$M_z = \sum_{i=1}^n m_i(v_{yi} x_i - v_{xi} y_i) = \text{const.} \quad (14)$$

That is why the relative change of the angular momentum

$$\delta M_z = (M_z - M_{z0})/M_{z0},$$

should be zero (here,  $M_{z0}$  is the value of the angular momentum at the initial time). If the angular momentum appears to be non-zero, the non-zero value should be considered as a consequence of some inaccuracy having occurred in numerical integration of the problem. More details about the inaccuracy of  $\delta M_z$  and the relation of this inaccuracy with the inaccuracies of coordinates and velocities can be found in [4, 5].

In January 2014 in the program Galactica the changes were made to the calculation of the relative change ( $\delta M_z$ ) of the projection of the angular momentum:

$$\delta M_z = (M_{za} - M_{za1})/M_{za1}, \quad (15)$$

where  $M_{za} = M_z + Sp_{sz}$  is  $z$ -projection of the aggregate angular momentum;

$Sp_{sz}$  is  $z$ -projection of the sum of the angular momenta due to self-rotation of bodies around their axes (spin projections);

$M_{za1}$  is  $z$ -projection of the aggregate angular momentum at the initial moment of time.

Non-zero values of the momenta  $P_x$ ,  $P_y$ , or  $P_z$  for the whole system may point to an occurrence of an inaccuracy in some particular direction. Extraordinarily high values of the maximum mass  $omm$  and velocity  $Um$  will suggest the user a likely source of inaccuracies.

### 9. Examples of input files galacf26.dat

The folder InpFiles contains examples of input files: such files can also be found in the subfolder Area. The input file galacf26.dat designed to solution of the problem of 3000 axisymmetric bodys interaction with a file of initial conditions axsymb3t.dat. An input file galacf26.v01 was used in solving the problem on multilayered ring structures [8]. The file FrtnColr.bmp gives the color numbers, from 1 to 12, adopted in standard Fortran graphics. Those colors can be set for displaying the velocity vectors of certain particles. As it was mentioned previously, the colors are set with parameters `ipp()` in the input file.

The folder Area gives six examples of input file galacf26.dat with setting initial conditions for interacting bodies obtained by partitioning a substance region into a certain number of bodies. Instead of the extension «dat», the names of the files contain symbols identifying the body system obtained. For each of the input files, a graphic bmp-file is also provided, whose name begins with the same symbols. Below we give a list of the problems represented with those files:

- 1) galacf26.3br – radial interaction involving three bodies;
- 2) galacf26.3b1 – three bodies with specified angular rate of rotation of the initial region;
- 3) galacf26.4bo – four bodies with specified rotational velocity;
- 4) galacf26.64r – radial interaction involving 64 bodies;
- 5) galacf26.64w – interaction of 64 bodies at specified rotational velocity of region;
- 6) galacf26.65b – interaction of 65 bodies, of which 64 bodies are represented with substance distributions in a region and the remaining body resides at a distance from the region.

Samples of input files galacf26.dat are available also in a folder InCndFls\Preprtn for start on the account of three samples of files of the initial conditions: axsymb30.dat, axsymb3c.dat and axsymb3t.dat in a graphic mode of the account.

### 10. Initial condition files for several solved problems

The folder InCndFls contains initial data and initial condition files for some solved problems. The data structure in those files was described in Sec. 5. Here, additional information about the use of such files is given.

The folder contains files of three types: 1) initial condition files; 2) body image files, named similarly to the initial condition files; 3) input files galacf26.dat, whose names are formed by symbols `g26` and by the names of the initial condition files. Files of the last type are presented only for several typical problems.

Files `sn49f.dat` and `sn49jplc.dat` were used to investigate into the evolution of the Solar system over a time interval of 100 million years [4, 5]. The files contain the masses, the coordinates, and the velocities of the planets, the Moon, and the Sun by the date 30.12.1949 with Julian day  $JD = 2433280.5$ . The data in the file `sn49jplc.dat` were borrowed from later Jet Propulsion Laboratory (JPL) ephemerides in comparison with `sn49f.dat`, namely, from DE406 ephemerides. That is why data in `sn49jplc.dat` more accurately reflect the properties of the Solar system. The files contain data for eleven bodies. In addition, a line for the twelfth body is reserved. To this line, data can be input for a flying vehicle or an asteroid whose motion in the Solar system is to be investigated.

The file sn69JPA2.dat contains initial conditions by the date June 28, 1969 with JD = 2440400.5. Apart from the mentioned bodies, this file additionally contains data for three asteroids: Ceres, Pallas, and Vesta, so that we have a total of fourteen bodies. The masses, the coordinates, and the velocities in that file were borrowed from the reference data of DE405 ephemerides [9]. In comparison with sn49f.dat and sn49jplc.dat, calculation by that file yields a better agreement of calculated data with the DE406 ephemerides for the planets from Jupiter to Pluto.

The files sn49e5n.dat, sn49e5n2.dat and sn49e5n3.dat contain data for three compound models (1-st, 2-nd, and 3-rd) of Earth rotation [4, 10]. The data for the eleven solar-system bodies precede the data for the bodies employed in the Earth rotation model. Their parameters are based on the same data as the parameters in sn49f.dat are, except for the Earth mass. In those files, the mass of the central body of the compound model of the Earth is reduced by the mass of the peripheral bodies considered in the model.

The files sn49S6J4.dat and sn49S6J5.dat contain data for the 4-th and 5-th compound models of the Sun [11-14]. Appearing first are data for the eleven solar-system bodies. Those data are based on the same sources as the parameters in sn49jplc.dat, except for the Sun mass. The mass of the central body in the Sun model is reduced by the mass of peripheral bodies involved in the model. Then, parameters of the peripheral bodies appear. In sn49S6J5.dat, the model involves ten bodies, and in sn49S6J4.dat, it involves five bodies.

The file sn49S6J6.dat contains initial data and initial conditions for the problem about the action of the compound Sun model on Mercury [15]. The file involves two objects, Mercury and the compound model of the Sun, the latter model involving a central body and ten peripheral bodies.

The files sn08AsAp.dat and sn08AsDA.dat [6, 7, 16-19] contain data about the eleven solar-system bodies and, in addition, asteroids Apophis and 1950 DA by the date 30.11.08 with Julian day JD = 2454800.5. The masses of the eleven solar-system bodies here are the same as in sn49jplc.dat. The coordinates and velocities of the bodies are based on the DE406 ephemerides. The coordinates and the velocities of the asteroids were calculated by an original procedure of [7] from the orbital elements of the asteroids borrowed from the NASA database of small bodies [20, 21].

In Sn2010Ap.dat, the data for the Solar system and Apophis are taken by the date 04.01.2010 with Julian day JD = 2445200.5. The masses are the same as in sn49jplc.dat. The coordinates and the velocity of Apophis were borrowed from the same sources as in previous files (see [7]).

The files snKo2102.dat and snKo2110.dat contain initial data and conditions for models of three-layered ring structures, the second and tenth ones, respectively [8, 22]. The mass of the body system is taken equal to the mass of the entire Solar system. The former file predicts a stable structure with the mass of the central body being equal to the Sun mass, and the latter file predict an unstable structure with the mass of the central body equal to half the Sun mass.

As noted above, all the parameters in the files are given in non-dimensional form: the masses are normalized by the mass  $M_{ss}$  of the body system in kilograms; the sizes, by the characteristic size  $A_m$  of the system in meters; and the velocity is multiplied by the coefficient  $k_v$  expressed in sec/m. For solar-system problems the non-dimensional time is expressed in sidereal centuries, and the barycentric equatorial coordinate system by the epoch 2000.0 is adopted. For sn49f.dat, the epoch 1950.0 is adopted. To the end of the file, an information line is appended indicating the date of creation of initial conditions, the number of involved bodies, and also the values of  $M_{ss}$ ,  $A_m$ , and  $k_v$ .

Apart from the above-listed files contained in folder InCndFls, other three initial condition files axsymb30.dat, axsymb3c.dat, and axsymb3t.dat are available; those files refer to axisymmetric interaction of 30, 300, and 3000 bodies, respectively. Those files, contained in folder Preprtn, can be used for testing Galactica codes. Besides in this folder there are the samples of input files galacf26.dat for start on calculation of the mentioned three files of the

initial conditions. The input file settings are for a good presentation of the results in graphical mode account. The parameters for good representation of results in graphic mode of the calculation are established in the input files.

### 11. Executable files

The folder RunFiles contains executable files. Table 2 lists the names of the main exe-files for PC and for a Unix OS supercomputer. In the names, the following agreements are adopted. Symbols glc mean Galactica. Digit 3 means a third version of the program intended for common use; the last number of the version in individual use was 34a. The character «b» with subsequent symbols means the total number of bodies.

Table 2. Names of main exe-files of Galactica

Number of interacting bodies	For PC		For a UNIX OS supercomputer
	Without graphics	With graphics	
300	glc3b3c.exe	glc3pb3c.exe	glc3b3c; glc3b3ce
30000	glc3b30t.exe	glc3pb30t.exe	glc3b30t

Executable files without graphics compiled by Fortran PowerStation v4.0. Codes with graphics, which are compiled by the Compaq Visual Fortran Professional Edition v6.5.0 in Fortran standard graphics, have in a name the letter p.

The code named glc3b3ce capable of calculating 300 bodies on a supercomputer operates with extended digit length (34 decimal symbols). This program can read initial condition files with double length numbers, and it generates integration data files with digits of quadruple length. The speed of calculation with this program is 33 times lower in comparison with other codes. It makes sense to use this version of Galactica in solving such problems as encounter of an asteroid with a celestial body or for calculating the motion of spacecrafts involving launch and landing trajectory portions, i.e. than a close approach of bodies is involved in the problem. All codes except for the latter one use double length numbers.

### 12. Generation of initial condition file

After getting acquainted with the preceding sections, the user is ready to tackle its own problem. In solving each problem, tasks may be encountered whose solution will require accomplishment of some other subtasks. A few such problems were solved in [1, 2, 4-8, 10-19, 22-23]. All problems having been resolved and the configuration of the body system and the initial motion, defined, the user can proceed with generation of an input data and initial condition file. As applied to the problem of axisymmetric interaction of  $N$  bodies [24], the file InCnPrpr.mcd exemplifies a procedure for generation of an initial condition file. The sample is written in MathCad software (see Appendix 1). This sample can be implemented using any programming language or mathematical software. Below, taking the sample as an example, we will comment on the particular actions. In the problem of interest, the following configuration of interacting bodies is defined: around a central body, peripheral bodies are located. The velocities of the bodies are chosen such that the bodies move along elliptic orbits with eccentricity  $e$ .

Item 12.1 of Appendix 1 defines the total number of interacting bodies  $N_b$  and the lengths of indexes  $i$  and  $i_1$ . Since an index begins with zero, the largest value of  $N$  is smaller than  $N_b$  by unity.

Item 12.2 defines the main constants, namely, the constant of gravitation  $G$ , the astronomical unit AU, the sidereal year in days  $P_{sd}$  and in seconds  $P_{sdsc}$ .

Item 12.3 defines the Earth mass  $m_{Er}$ , the Sun mass  $M_S$ , the mass of the Solar system  $M_{Ss}$ , the body densities  $Roba$ , the major semi-axis of the orbit of the peripheral bodies ( $a$ ), the eccentricity of the orbit ( $e$ ), and the inclination  $\varepsilon$  of the orbit plane to the  $xy$ -plane, which is assumed to be the Earth's equatorial plane.

In 12.4, for all the interacting bodies, masses in kilograms and radii in meters are calculated.

Then, in item 12.5 the algorithm of [1, 2, 24] is implemented to determine the coordinates and velocities of the peripheral bodies in the orbit plane. Here, the notation introduced in our

book «The Theory of Interaction» [1-2] is used:  $\alpha_1$  is the trajectory parameter,  $\mu_1$  is the interaction parameter, and the pericenter parameters are  $R_p$  and  $v_p$ :  $R_p$  being the radius and  $v_p$ , velocity.

This algorithm is intended for calculating the interaction of an arbitrary number of bodies. That is why one has to check that the orbit will not be overcrowded with the bodies. This check can be performed as follows: the distance between two bodies do reduced by the sum of their radii should be more than 50. Otherwise in Sec. 12.1 the major semi-axis value “a” should be increased using the coefficient  $k_a$ .

The problem of axisymmetric interaction of bodies was solved exactly in [1, 2, 24]. Here, the peripheral bodies move along ellipses. In item 12.6 those orbits are calculated, and they are plotted in coordinates  $y_{nb}-x_{nb}$ . The index  $i2=1,3.. N$  in MathCad can be used to call for plotting graphs not for all bodies. Here, orbits are to be plotted for each third body. Those graphs confirm that the set configuration of interacting bodies is indeed axisymmetric, and the bodies move in an elliptic orbit with the required eccentricity  $e$ .

In item 12.7, the orbit plane rotates through an angle  $\varepsilon$  relative to the axis  $x$ . Here, differential equations (6) «work» along all the three axes,  $x$ ,  $y$ , and  $z$ , despite the fact that the initial statement of the problem was planar. Due to the latter rotation, to 20 trustworthiness criterions considered in [4, 5], one more criterion can be added. If integration of Eqs. (6) yields orbits deflected from the initial plane, then the deviations should be identified as inaccuracies of the integration method.

In item 12.8, we pass to non-dimensional variables. Here,  $ms_{N+1}$  is the sum of the masses of all the bodies involved in the system,  $Am$  is the scaling length factor in meters, and  $k_t$  and  $k_v$  are the scaling time and velocity factors expressed in SI units. Those factors were described in Sec. 2.

In the latter example, the scaling factor  $Am$  is calculated from the condition that the non-dimensional time unit should be equal to sidereal century  $100 \cdot P_{sd}$ . Since  $Am$  is determined with some non-zero inaccuracy due to extraction of a cubic root, then a procedure for refining the quantity  $Am$  is to be applied.

In item 12.9, according to formulas (4)-(5) the variables are reduced to the center of mass of the system. Here, for instance,  $X_{c_{N+1}}$  and  $V_{x_{c_{N+1}}}$  are the coordinate and the velocity of the center of mass. In the problem of interest, initially the center of mass is at the origin, so that the above values are both close to zero. Nonetheless, the procedure should be accomplished, and the parameter values of the center of mass, examined. The latter examination is a check of the algorithm for calculating initial conditions. In the present case, the values of the parameters are within  $4 \cdot 10^{-19}$ . That is why for performing Galactica calculations with a lower error preparation of initial conditions with a better accuracy is required. For instance, it would be desirable that the initial conditions be prepared with a larger number length than it can be made in MathCad.

In item 12.10, projections of the angular momentum  $M_{x_{N+1}}$ ,  $M_{y_{N+1}}$ , and  $M_{z_{N+1}}$  of the whole body system and the absolute value of the angular momentum  $M_0$  are calculated. For the problem of interest, the total angular momentum of the whole system can also be calculated as

$$M_0 = (Nb - 1) \cdot m_1 \cdot R_p \cdot v_p. \quad (16)$$

Then, the check is to be performed. The sum of non-dimensional masses of the system should equal unity. We determine the relative error of the angular momentum  $M_0$  and its value  $M_0$  calculated by formula (16), and also the relative inaccuracy of the scaling factors  $Am$ ,  $k_t$ , and  $k_v$ . Those checks enable evaluation of the inaccuracy of the algorithm, and also the value of the computational inaccuracy. As it is seen, here the residual of the scaling factors is zero. This result was obtained through refinement of  $Am$  in item 12.8.

In item 12.11, we calculate the number  $Mu$  to be used in the input file galacf26.dat for specifying initial conditions for space-distributed substance. Here, the parameter  $Mu$  is not used. However, for right representation of bodies in graphic mode this number should be made consistent with the number of bodies  $Nb$ . The table below exemplifies matched parameters for different values of  $Nb$ :



B	C	Mu	Nb
0.5	0.25	4	30
0.9	0.6	7	300
0.9	0.7955	15	3000

In item 12.12, initial data and conditions are written in a prn-file, for instance, ax symb30b.prn. The file contains 15 columns. The first two lines contain general data involving 24 parameters. That is why six zeroes at the end of the second line are to be removed, and the file is to be renamed in a dat-file, for instance, ax symb30b.dat.

After the first two lines, additional lines, each comprising 15 values, appear. The total number of the lines is equal to the total number of the bodies involved in the problem. At the end, there appears an information line, whose first number is the date of creation of the initial condition file: year.mm.dd.

The next two graphs show the coordinates and velocities in the body system. Those values enable a check of the sought configuration planned in the problem statement. As it is seen from the graphs, the coordinates and velocities of the bodies form an axisymmetric pattern; in other words, they correspond to the problem of axisymmetric interaction of the bodies.

The above-considered program InCnPrpr.mcd was used to generate the following files: ax symb30.dat, ax symb3c.dat, and ax symb3t.dat for 30, 300, and 3000 bodies, respectively. Those files permit check of operability of the exe-files, evaluation of the total errors in solving the problem, and integration step adjustment ensuring a required accuracy of problem solution.

At the end of the sample program, two more points are outlined which may appear necessary is setting initial conditions. In item 12.13, an algorithm for calculating the number of steps Ltk and the corrected time step value dTc for exact solution of the problem from one date with Julian day JD1 to another date with Julian day JD2 are considered. The need in the calculation arises in solving problems on interaction of solar-system bodies.

If the non-dimensional time in Galactica calculations is expressed in sidereal centuries, then the time difference  $\Delta T_{jd}$  in Julian days is transformed into the number  $\Delta T$  in sidereal days. Given the step dT, we calculate the number of steps Ltkr to be rounded to a close value Ltk. Then, the corrected step dTc is to be determined.

Here, the calculations were performed for two variants: (i) in resumed calculation mode (with key K13 set to 1 or 3), and (ii) in initial calculation mode (with key K13 set to 2 or 4). In the latter case, as it was mentioned previously, the calculation at the first step is to be performed with the step  $0.001 \cdot dT$ .

At the end, check of the calculated values of Ltk and dTc is to be performed.

Item 12.14 gives an algorithm for reading out an initial condition file or an integration data file of type gazez28.dat. This becomes needed in setting initial conditions from a calculated data file of an analogous problem. Besides, that algorithm can be used for analyzing the output data.

In the first line, the number of bodies Nbg and index lengths ig and ig2is are defined. In the first line, the number of bodies is set. Here, instead of the operator READPRN, the operator READ is used. The latter operator permits reading-out of a non-ordered file. After the reading, required quantities can be displayed as shown in the graph.

### 13. Problem solving procedure

After the user has made his decision concerning the configuration of the system of interacting bodies, their masses, sizes, and initial conditions, including the coordinates and velocities of the bodies, he has to proceed with generation of an initial condition file. For passage to non-dimensional variables, one has to choose, in addition to the mass of the system  $M_{ss}$ , one more scaling factor, namely, the distance Am or a non-dimensional time unit. In the latter case, the quantity Am will be defined using the non-dimensional time.

Next, in accordance with Sec. 12, one has to produce an initial condition file, for instance, a file named initial.dat. In generating the latter file, one has to introduce into the

algorithm, where possible, all checks (in Sec. 12, several such checks were outlined), so that to achieve full compliance of file parameters with the determined configuration of the problem. As it was shown in Sec. 12, it would be highly desirable to output the configuration of body positions and velocities in graphic form. With a large number of involved bodies, the latter offers a most reliable checking strategy. Then, one has to generate numerical values for the input file galacf26.dat. In Sec. 4, a description of the parameters entering that file is given and their functional meaning is indicated. For first starts of the problem, only necessary functions are to be used. Subsequently, as may be necessary, other functions, one by one, can be added.

Then, the user is invited to choose an exe-file from folder RunFiles. For instance, if, in the system of interest, the total number of interacting bodies is  $k_2 = 29$ , then the exe-file intended for 30 bodies is to be used. An exe-file intended for treating a greater number of bodies can also be employed. Yet, such an exe-file will use more RAM memory space; in some cases this circumstance will slow down the solution of the problem.

If the user solves a problem with the help of Galactica for the first time, a most appropriate strategy would be to choose from the folders InCndFls, Preprtn, and InpFiles a sample input file and a sample initial condition file, and then test the operation of the chosen exe-file in various modes of operation.

Then, the user may start its own problem: first over one step and, then, over a few steps. Here, errors in reading-out of the input file or the initial data file can be encountered. In the latter case, the user must use another program, e.g. the program InCnPrpr.mcd, to read out the files with the operator READ, and get output the values of all parameters. As a rule, errors in reading out the input file can be identified through comparison with a similar readable file.

If the running time of the program is long, the user has to perform calculations over short running time to determine the time required for executing one step and, then, he has to evaluate the time required for solving the whole problem. Then, the user has to calculate, using the formulas given in Secs. 5 and 6, the total volume of output files: 1, 2, 3 ... garez28.dat, and 1t, 2t, 3t ... traekt.prn. It is also necessary to check that this volume does not exceed the volume allocated for the solution of the problem.

After the start of the exe-file, through observation of data on the display screen or data issued in the output stream file (see Fig. 1) the user can monitor the solution process. For the user, each of the 20 issued parameter described in Sec. 8 can serve an indicator of the specific features of the problem. The behavior of those parameters can be helpful in revealing an error in the problem statement or, on the contrary, this behavior can justify the problem statement, or even help in predicting the final result.

For analysis, data should be represented in graphic form. To be analyzed is the variation of the coordinates of individual bodies in time, i.e. the trajectories of the bodies. If the trajectories are cyclic, then the orbit elements of bodies should be identified, and the evolution of elements in time, analyzed.

In our previous publications, a number of methods and programs for analyzing calculated data were developed. In the future, we are going to adapt the programs for general use and apply them to the Galactica system.

## **14. Module of system Galactica with Coulomb's interaction**

### ***14.1. Introduction***

#### ***14.1.1. Area of application for module with Coulomb's interaction***

In physics Coulomb's interaction usually is considered on the basis of the two-body problem. In quantum mechanics behavior of ensembles of particles and their properties are studied as a result of statistical processing of two-particle interactions. Together with such quantum-mechanical consideration of the microworld, a number of researchers continue to apply classical mechanics for explanation of a series of its phenomena. A.D. Vlasov [25] in his researches has concluded about justice of the classical electrodynamics laws in atom and about inability of probabilistic interpretation of atomic phenomena. F.M. Kanarev [26]-[27] explains atoms radiation spectra on the basis of classical physics. During several decades M. Gryziński

consistently considers the microworld phenomena, based on the Coulomb's mechanism of interaction. For example, he explains diffraction phenomena by electron spin precession [28]. Using binary Coulomb's interactions, M. Gryziński explains single and double ionization, radiation of one or triplet lines, diffraction of particles at their dispersion on atoms and molecules [29]. He has showed that taking into account of action of atom electron shell explains Ramsauer's effect about low dispersion of electrons at their low energy [30]-[31]. On the basis of classical mechanics M. Gryziński has received equations for determination of absolute braking energy of particles by any medium in all nonrelativistic spectrums of energies [32].

The above-mentioned researchers obtained the enumerated results by analytical methods. However, all these tasks are difficult, and only separate problems of many particles interaction can be solved even at high mathematical level of a researcher. Therefore use of high-precision numerical methods of solution of these problems of Coulomb's interaction opens a prospect of determined knowledge of the microworld.

As shown in works [1], [2], [33], the essential change of particles trajectories happens at velocities of motion  $v > 0.1c$ , where  $c$  is speed of light. Therefore it is possible to investigate the phenomena unto velocities of this order by system Galactica with Coulomb's interaction. It is necessary to use exact expression for the force of electromagnetic interaction of two particles [1] in the case of higher velocities. Further we hope to update system Galactica by the module for calculation of electromagnetic interactions at high velocities which are now called relativistic ones.

#### *14.1.2. Essentials of the module*

The module is intended for solving problems in mechanical interaction of material-point particles according to the Coulomb's law. Module files are located in folder GalactW/ModCoulm. This folder contains other folders: InCndFsQ, InpFilsQ and RunFilsQ. The folder GalactW contains files GalDiscrp.pdf and GalDiscrE.pdf that are manual of Galactica respectively in Russian and in English. The zipped folders can be downloaded at <http://www.ikz.ru/~smulski/GalactW/ModCIFlds.zip>.

The system includes named executable files, for instance, glk3b30e, to be run on a supercomputer under Unix OS, or glk3b30.exe, to be run under DOS; an input file galacf26.dat, and a named initial data file specifying the interacting bodies and the initial conditions of their motion, for instance, axsykl09.dat. Below, the latter file will be referred to as the initial condition (IC) file.

The results yielded by the executable file are produced as output files. The representation form is defined by keys specified in the input file galacf26.dat. Following a certain number of steps, characteristics of body motions can be issued; files containing such data are named 1, 2, 3... garek29.dat. The file garek29.dat is always produced on completion of the program. This file contains characteristics of bodies at the final calculation step. This is the main form of problem solution output.

Besides, if certain keys are set in the input file galacf26.dat, then parameters defining the trajectory of particles, whose numbers are also defined in the input file, are issued too. The names of the trajectory files are as follows: 1t, 2t, 3t ... traekt.prn. The file size is defined by the volume of the data set in the input file.

If certain keys are set, then a file dice.dat is issued. The file contains data on the time of close approach of bodies, minimal distance between bodies, and the number of a body approached by the given body. Each line in the file dice.dat is formed on accomplishment of a certain number of integration steps. The number of such steps is specified in the file galacf26.dat.

With a certain key set, a binary file (or, depending on the particular computer, an unformatted file) named galgok2 is issued. This file contains the state of the problem in binary form at the last calculation step. On subsequent start of the program, calculation is continued starting from this step, the data contained in the input file and in the initial condition file not

being read out. The latter procedure is required in solving problems with long running times. The interruptions in calculations do not worsen the solution accuracy.

In the case of an emergency situation, an error file ErrGletca is generated; this file contained a description of the error having occurred. Such errors can be encountered in opening files and their reading, and they also can be a result of insufficient array dimensions or impossibility to meet accuracy requirements in calculation of bodies merging together on collision.

In Galactica, the possibility to produce a screen of data is provided. Those parameters common to all bodies allows one to monitor both the execution of the program and the calculation accuracy. In solving a problem on a supercomputer, such data are saved in an output stream file.

In a version of Galactica with PC display graphics, along with common problem parameters, images of bodies with their velocity vectors can be produced. The form of images is defined by keys and parameters set in the input file galacf26.dat.

#### 14.2. Differential equations of motion for material-point particles

Let there is a system of  $N$  material points (particles) with masses of  $m_i$  and electrical charges  $q_i$ , where  $i = 1, 2 \dots N$ . Let's designate dimensional coordinates and velocities of particle  $i$  as  $x_{mi}, y_{mi}, z_{mi}, v_{xmi}, v_{ymi}, v_{zmi}$  in the non-accelerated system of coordinates relative to the center of mass  $C$ . In particular, the coordinates can be expressed in meters and velocities in m/sec. A particle with number  $k$  acts on  $i$ -th particle by the electrostatic action, which will be written down as Coulomb's force projection onto the axis  $x_m$  so:

$$F_{xik} = \frac{q_i q_k (x_{mi} - x_{mk})}{\varepsilon_d \cdot r_{mik}^3}, \quad (17)$$

where  $\varepsilon_d$  is dielectric permeability of medium, in which the particles are situated;

$$r_{mik} = \sqrt{(x_{mi} - x_{mk})^2 + (y_{mi} - y_{mk})^2 + (z_{mi} - z_{mk})^2} \quad (18)$$

is the distance between the  $i$ -th and  $k$ -th particles.

The expression (17) for force projection onto the axis  $x_m$  is written down in CGS electrostatic system. The formula for force projection onto the axes  $y_m$  and  $z_m$  looks similarly. Therefore all expressions will be written down here and below for one projection. With summation of all particles forces (17), the Coulomb's force of their action upon the particle with number  $i$  will be

$$F_{xi} = q_i \sum_{k \neq i}^N \frac{q_k (x_{mi} - x_{mk})}{\varepsilon_d r_{mik}^3}, \quad (19)$$

where summation for  $k = 1, 2 \dots N$ , except  $k = i$ , is designated by symbol  $\Sigma$ .

Under the action by force (19) the  $i$ -th particle will set in motion relative to an non-accelerated reference system with acceleration

$$\frac{dv_{xmi}}{dt} = \frac{q_i}{m_i} \sum_{k \neq i}^N \frac{q_k (x_{mi} - x_{mk})}{\varepsilon_d r_{mik}^3}. \quad (20)$$

Below the problem is solved in non-dimensional form. Characteristic size  $Am$  of an area, in which there are electric particle, is set for it. All charges  $q_i$  are reduced by absolute value of electron charge  $e_e$ , which  $e_e = +4.80298 \cdot 10^{-10} \text{ cm}^{3/2} \cdot \text{g}^{1/2} \cdot \text{sec}^{-1}$ , for example, in CGS electrostatic system. The masses  $m_i$  are reduced by total mass  $M_{SS}$  of all system of interacting particles. Then equation (20) will look in non-dimensional form so:

$$\frac{dv_{xi}}{dT} = -q_{mi} \cdot \sum_{k \neq i}^N \frac{q_{ok} \cdot (x_i - x_k)}{R_{ik}^3}, \quad i = 1, 2, \dots, N, \quad (21)$$

where  $x_i = x_{mi} / Am$ ;

$$R_{ik} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2}; \quad (22)$$

$$q_{oi} = q_i / e_e; \quad q_{mi} = -q_{oi} / m_{oi}; \quad (23)$$

$$m_{oi} = m_i / M_{SS}; \quad M_{SS} = \sum_{i=1}^N m_i; \quad (24)$$

$$v_{xi} = v_{mxi} \cdot k_v; \quad k_v = \sqrt{\frac{\varepsilon_d M_{SS} \cdot Am}{e_e^2}}; \quad (25)$$

$$T = t \cdot k_t; \quad k_t = \sqrt{\frac{e_e^2}{\varepsilon_d \cdot M_{SS} \cdot Am^3}} = 1/(Am \cdot k_v). \quad (26)$$

Expression (21) for three projections  $x$ ,  $y$  and  $z$  represents  $3 \cdot N$  second order differential equations, which determine motion of the charged particles. Non-dimensional charges  $q_{oi}$  of particles can be positive and negative. Value of non-dimensional variables, which describe motion by equations (21), depends on arbitrary parameter  $Am$ . Its value can be chosen such that non-dimensional time  $T$  has been in convenient units for consideration of interactions in the microworld.

If the primary scale is time coefficient  $k_t$ , then the scale length  $Am$  entering the expression (26) for  $k_t$  can be calculated as

$$Am = \left( \frac{e_e^2}{\varepsilon_d \cdot M_{SS} \cdot k_t^2} \right)^{1/3}. \quad (27)$$

Gravitational interactions in program Galactica are determined by the following non-dimensional equation (6) in [34]. Let's copy it without changes:

$$\frac{dv_{xi}}{dT} = - \sum_{k \neq i}^N \frac{mo_k (x_i - x_k)}{r_{ik}^3}, \quad i = 1, 2, \dots, N. \quad (28)$$

It is visible from comparison (21) with (28) that the algorithm for gravitational interaction can be adapted to Coulomb's one if, first, to replace  $mo_k = m_{ok}$  with  $q_{ok}$  and, secondly, to multiply expression (28) for acceleration  $dv_{xi}/dT$  by  $q_{mi}$ . These changes were done in program Galactica, and Fortran-program glk3pb30.for for Coulomb's interaction is created [35].

### 14.3. Solution method

This item is identical to section 3. **Solution method**

### 14.4. Input file galacf26.dat

This item is identical to section 4 with the identical name.

Unlike gravitational interactions, the possibilities of item 4.2. for Coulomb's interaction aren't used. Besides, the name of the binary file galgok2 is used instead of name galgon4.

### 14.5. Initial condition and integration data files

Section 5 with the identical name, is basis for this item. Some differences for Coulomb's interaction are given below.

Each of initial condition files, for example, axsykl09.dat in folder InCndFsQ, and also output files: 1, 2, 3, etc. and file garek29.dat begin with a set of 25 numbers: T0, omm, Um, dTp, Px, Py, Pz, AMx, AMy, AMz, Spsx, Spsy, Spsz, E, Em, Ett, dT, i2b, j2b, k2b, A, B, C, Mu, ed. Those numbers are common to all interacting particles. Then, sixteen numbers for each  $k$ -th particle follow: om(k), (X(k,q),q=1,3), (U(k,q),q=1,3), (dUp(k,q),q=1,3), (Sp(k,q),q=1,3), Ra(k), Et(k), qo(k). Here,  $q$  is the subscript listing the variables over the three coordinates  $x$ ,  $y$ , and  $z$ .

The general parameters represent the following:

T0 – time in non-dimensional units;

omm – maximal mass of body reduced by the total mass of all particles;

Um – maximal velocity in non-dimensional units;

dTp – time step in non-dimensional units at the previous step;

Px, Py, Pz – projections of the total momentum of the whole particle system on the coordinate axes;

AMx, AMy, AMz – projections of the total angular momentum of the whole particle system;

Spsx, Spsy, Spsz – projections of the sum of the angular momenta due to self-rotation of bodies around their axes (spin projections);

E – total kinetic energy of all particles at the given moment T0;

Em – maximal kinetic energy of all particles starting from the beginning of integration;

Ett – thermal energy of all bodies which the particles have acquired during their merging on collision;

dT – current time step in non-dimensional units;

ed – dielectric permeability of medium, which there are the particles in.

Next, 16 parameters for each particle follow:

om(k)  $\div m_{ok}$  – non-dimensional mass of the k = i-th particle (k and i are the numbers of particles in different texts);

(X(k,q),q=1,3)  $\div x_i, y_i, z_i$  – non-dimensional barycentric coordinates of the k = i-th particle;

(U(k,q),q=1,3)  $\div v_{xi}, v_{yi}, v_{zi}$  – non-dimensional velocities of the k = i-th particle;

(dUp(k,q),q=1,3)  $\div x_i^{(5)}, y_i^{(5)}, z_i^{(5)}$  – non-dimensional derivatives of the fifth order for the k = i-th particle;

(Sp(k,q),q=1,3)  $\div S_{pxi}, S_{pyi}, S_{pzi}$  – non-dimensional angular momenta (spins) due to the self-rotation of the k = i-th particle. A particle is assumed to get a spin if the particle has formed as a result of the merge of other particles having approached each other to a distance smaller than the sum of their radii. Initially, the rotation of particles can be ignored, with their spins being defined as  $S_{pxi}=S_{pyi}=S_{pzi}=0$ . This strategy is adopted in the initial condition files given below.

Ra(k) – non-dimensional equivalent radius of the k = i-th particle;

Et(k) –  $E_{ti}$  – non-dimensional thermal energy of the k = i-th particle. This energy has arisen from merging of several particles in one particle due to transformation of kinetic energy into thermal energy.

qo(k) – non-dimensional charge of the k = i-th particle.

In Appendix 3 there is the sample of program InCnPrClb.mcd in MathCad software for generation of initial condition file.

#### 14.6. Trajectory files 1t, 2t ... traekt.prn

This item is identical to section 6.

#### 14.7. Close-approach file dice.dat

This item is identical to section 7.

#### 14.8. On-screen display

This item is identical to section 8. Similarly Fig. 1 in section 8 for gravitational interaction, Fig. 2 yields output for Coulomb's interaction [35].

```

1.0000000000000000E-010  0.999727944522533      307.702140230615
1.0000000000000000E-010 -8.673617379884035E-019  8.673617379884035E-019
-4.336808689942018E-019  8.673617379884035E-019 -5.371593013823018E-002
0.123928527828917      0.0000000000000000E+000  0.0000000000000000E+000
0.0000000000000000E+000  12.8791888860771      12.8791888860771
0.0000000000000000E+000      1  1.0000000000000000E-007
0.0000000000000000E+000

```

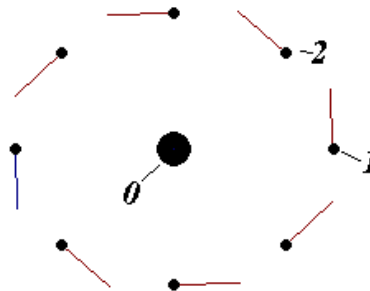


Fig. 2. Axisymmetric Coulomb's interaction between 9 particles with parameters for oxygen atom: 0 – the central particle; 1 – the first peripheral particle; 2 – the second peripheral particle. Case of circular motion.

### 14.9. Examples of input files galacf26.dat

Section 9 gives samples of input files for different problems of gravitational interaction. These samples show functions of various parameters of file galacf26.dat.

The folder InpFilsQ contains example of input file galacf26.dat, which was used for axisymmetric Coulomb's interaction of 9 particles (see Fig. 2) in work [35]. This folder also contains input files with changed names g26\_2ptk, g26\_3ptk, g26\_175ptk, g26\_473ptk for problems with 2, 3, 175 and 473 particles. Files with 2 and 3 particles were used in work [35]. The force of attraction between a peripheral particle and the central particle is maximal for axisymmetric configuration with 175 particles, and minimal for one with 473 particles.

### 14.10. Initial condition files for several solved problems

Section 10 involves initial condition files for various problems of gravitational interaction. The folder InCndFsQ contains file ax8ykl09.dat for axisymmetric Coulomb's interaction of 9 particles. Thus 8 peripheral particles move along elliptic orbits with eccentricity  $e = 0.15$ . This folder also contains initial condition files with analogical names for interaction of 2, 3, 175 and 473 particles. Their orbits are circles. The program InCnPrC1b.mcd, contained in this folder, is intended for creation of such initial condition files.

### 14.11. Executable files

In section 11 the executable files, which are in the folder RunFiles, are described in the case of gravitational interaction. The folder RunFilsQ contains executable files in the case of Coulomb's interaction. Table 3 lists the names of the main exe-files for PC and for a Unix OS supercomputer. In the names, the following agreements are adopted. Symbols glk mean Galactica with Coulomb's interaction. Digit 3 means a third version of the program intended for common use. The character «b» with subsequent symbols means the total number of bodies (particles).

Table 3. Names of main exe-files of Galactica with Coulomb's interaction

Number of interacting particles	For PC		For a UNIX OS supercomputer
	Without graphics	With graphics	
30	–	–	glk3b30e
300	glk3b3c.exe	glk3pb3c.exe	–
3000	glk3b3t.exe	glk3pb3t.exe	glk3b3t

Executable files without graphics compiled by Fortran PowerStation v4.0. Codes with graphics, which are compiled by the Compaq Visual Fortran Professional Edition v6.5.0 in Fortran standard graphics, have in a name the letter p.

The code named glk3b30e capable of calculating 30 particles on a supercomputer operates with extended digit length (34 decimal symbols). This program can read initial condition files with double length numbers, and it generates integration data files with digits of quadruple length. The speed of calculation with this program is 33 times lower in comparison with other codes. All codes except for the latter one use double length numbers.

### 14.12. Generation of initial condition file

In section 12 the recommendations for generation of initial condition file, the part of which can be used in the case Coulomb's interaction, are given in the case of gravitational interaction. As it is noted in section 12, there are features of algorithm for calculating initial conditions for each task. As applied to the problem of axisymmetric Coulomb's interaction [35], the file InCnPrC1b.mcd (see Appendix 3) exemplifies a procedure for generation of an initial condition file. This sample can be implemented using any programming language or mathematical software. Below, taking the sample as an example, we will comment on the particular actions. In the problem of interest, the following configuration of interacting particles is defined: around a central particle, peripheral particles are located. The velocities of the particles are chosen such that the particles move along elliptic orbits with eccentricity  $e$ . The

central particle has a positive charge, and peripheral ones have a negative charge. Such problem has an exact analytical solution. In works [1], [2] and [24] it is given for gravitational interaction, and in work [35] – for Coulomb's one. Such axisymmetric configurations are unstable at certain parameters. Therefore in the case of numerical solution of the particles interaction equations, their dynamic features are determining. In work [35] the interaction between the central particle and eight and two peripheral particles, and also interaction between two particles are considered. The program InCnPrClb.mcd was used for generation of initial condition file for these tasks. Here it also was used for generation of initial condition file for these tasks to the number of particles 175 and 473.

Item 14.12.1 of Appendix 3 defines the total number of interacting bodies  $N_b$  and the lengths of indexes  $i$  and  $i1$ . Since an index begins with zero, the largest value of  $N$  is smaller than  $N_b$  by unity.

Item 14.12.2 defines the main constants, according to [36], namely, masses of electron  $m_e$ , proton  $m_p$  and neutron  $m_n$  in kg; charge of electron  $e_e$  in CGS electrostatic system; base radius  $R_0$ , which atom nucleus radius is determined by

$$R_n = R_0 \cdot A_N^{1/3}, \quad (29)$$

where  $A_N$  is mass number of an atomic nucleus.

Besides, the item 14.12.2 defines electron radius  $R_e$ , dielectric permeability of medium  $\varepsilon_d$  in CGS electrostatic system and the first Bohr's orbit radius  $a_{B0}$ , by which the average distance between electron and nucleus (see p. 749 [36]) is determined:

$$r_m = \frac{a_{B0}}{2 \cdot Z_p} [3 \cdot n_n^2 - l_n(l_n + 1)], \quad (30)$$

where  $Z_p$  is charge of an atomic nucleus;

$n_n$  and  $l_n$  are quantum numbers.

Item 14.12.3 defines axisymmetric configuration with 8 peripheral particles by the example of oxygen atom, including atomic number  $A_N$ , nucleus charge  $Z_p$  and quantum numbers  $n_n$  and  $l_n$  for determination of average distance  $a = r_m$  between peripheral electrons ring and nucleus by formula (30). This item also defines the orbits eccentricity of peripheral particles and the plane inclination  $\varepsilon_0$  of their orbits  $x_0y_0$  in the coordinate system  $x y z$ , for which the equations (21) are written. The orbits plane  $x_0y_0$  is revolved through an angle  $\varepsilon_0$  relative to the axis  $x$ .

In item 14.12.4 the masses of the central particle  $m_0$  and the peripheral particles  $m_{i1}$  in kilograms, and also their radii  $ram_0$  and  $ram_{i1}$  in meters are calculated.

Then, in item 14.12.5 the algorithm of [1, 2, 24, 34] is implemented to determine the coordinates and velocities of the peripheral particles in the orbit plane. Here, the notation introduced in our book «The Theory of Interaction» [1-2] is used:  $\alpha_1$  is the trajectory parameter,  $\mu_1$  is the interaction parameter, and the pericenter parameters are  $R_p$  and  $v_p$ :  $R_p$  is the radius and  $v_p$  is velocity.

This algorithm is intended for calculating the interaction of an arbitrary number of particles. That is why one has to check that the orbit will not be overcrowded with the particles. This check can be performed as follows: the distance between two particles divided by the sum of their radii should be more than 50. Otherwise in Sec. 12.1 the major semi-axis value “a” should be increased using the coefficient  $k_a$ .

The problem of axisymmetric interaction of bodies was solved exactly in [1, 2, 24, 34]. Here, the peripheral bodies move along ellipses. In item 14.12.6 those orbits are calculated, and they are plotted in coordinates  $y_n b(x_n b)$ . The index  $i2=1,2.. N$  in MathCad can be used to call for plotting graphs not for all particles. For example, the orbits will to be plotted for each third particle at the index  $i2=1,3.. N$ . Those graphs confirm that the set configuration of interacting particles is indeed axisymmetric, and the particles move in an elliptic orbit with the required eccentricity  $e$ .

In item 14.12.7, the orbit plane rotates through an angle  $\varepsilon$  relative to the axis  $x$ . Here, differential equations (21) «work» along all the three axes,  $x$ ,  $y$ , and  $z$ , despite the fact that the



initial statement of the problem was planar. Due to the latter rotation, to 20 trustworthiness criterions considered in [4, 5], one more criterion can be added. If integration of Eqs. (21) yields orbits deflected from the initial plane, then the deviations should be identified as inaccuracies of the integration method.

In item 14.12.8, we pass to non-dimensional variables. Here,  $ms_{N+1}$  is the sum of the masses of all the particles involved in the system,  $Am$  is the scaling length factor in meters, and  $k_t$  and  $k_v$  are the scaling time and velocity coefficients expressed in SI units. Those parameters were described in Sec. 2 and 14.2.

In the latter example, the scaling factor  $Am$  is calculated from the condition that the non-dimensional time unit should be equal to  $P_m = 1 \cdot 10^{-15}$  sec. Since  $Am$  is determined with some non-zero inaccuracy due to extraction of a cubic root, then a procedure for refining the quantity  $Am$  is to be applied.

It should be noted that ratio  $2 \cdot a / Am = 3.23$  for given unit of time  $P_m$ , i.e. the order is close to 1. In this case the scaling factor  $Am$  is a characteristic size of a considered system, namely an average size of orbit  $2 \cdot a$ .

In item 14.12.9, according to formulas (4)-(5) the variables are reduced to the center of mass of the system. Here, for instance,  $Xc_{N+1}$  and  $Vxc_{N+1}$  are the coordinate and the velocity of the center of mass. In the problem of interest, initially the center of mass is at the origin, so that the above values are both close to zero. Nonetheless, the procedure should be accomplished, and the parameter values of the center of mass, examined. The latter examination is a check of the algorithm for calculating initial conditions. In the present case, the values of the parameters are within  $5 \cdot 10^{-18}$ . That is why for performing Galactica calculations with a lower error preparation of initial conditions with a better accuracy is required. For instance, it would be desirable that the initial conditions be prepared with a larger number length than it can be made in MathCad.

In item 14.12.10, projections of the angular momentum  $Mx_{N+1}$ ,  $My_{N+1}$  and  $Mz_{N+1}$  of the whole particle system and the absolute value of the angular momentum  $M0$  are calculated. For the problem of interest, the total angular momentum of the whole system  $M00$  can also be calculated by formula (16). Then, the check is to be performed. The sum of non-dimensional masses of the system should equal unity. We determine the relative error of the angular momentum  $M0$  and its value  $M00$  calculated by formula (16), and also the relative inaccuracy of the scaling factors  $Am$ ,  $k_t$  and  $k_v$ . Those checks enable evaluation of the inaccuracy of the algorithm, and also the value of the computational inaccuracy. As it is seen, here the residual of the scaling factors is zero. This result was obtained through refinement of  $Am$  in item 14.12.8.

In item 14.12.11, we calculate the number  $Mu$  to be used in the input file galacf26.dat for specifying initial conditions for space-distributed substance. Here, the parameter  $Mu$  is not used. However, for right representation of particles in graphic mode this number should be made consistent with the number of particles  $Nb$ . The table below exemplifies matched parameters for different values of  $Nb$ :

B	C	Mu	Nb
0.5	0.25	4	30
0.9	0.6	7	300
0.9	0.7955	15	3000

In item 14.12.12, initial data and conditions are written in a prn-file, for instance, axsykl09.prn. The file contains 16 columns. The first two lines contain general data involving 25 parameters. That is why one zero at the end of the first line is to be removed, and six zeroes at the end of the second line are to be removed, and the file is to be renamed in a dat-file, for instance, axsykl09.dat.

After the first two lines, additional lines, each comprising 16 values, appear. The total number of the lines is equal to the total number of the particles involved in the problem. At the end, there appears an information line, whose first number is the date of creation of the initial condition file: year.mm.dd.

The next two graphs show the coordinates and velocities in the particle system. Those values enable a check of the sought configuration planned in the problem statement. As it is seen from the graphs, the coordinates and velocities of the particles form an axisymmetric pattern; in other words, they correspond to the problem of axisymmetric interaction of the particles.

The above-considered program InCnPrpr.mcd was used to generate the following files: axsyk175.dat and axsyk473.dat for 175 and 473 particles, respectively. Those files permit check of operability of the exe-files, evaluation of the total errors in solving the problem, and integration step adjustment ensuring a required accuracy of problem solution. These axisymmetric configurations are noticeable by the maximal force of attraction between a peripheral particle and the central particle for the 175 system particles, and minimal – for the 473 system particles. If there are more particles in configuration, that peripheral particles are rejecting from the center.

Item 14.12.13 gives an algorithm for reading out an initial condition file or an integration data file of type garek29.dat. This becomes needed in setting initial conditions from a calculated data file of an analogous problem. Besides, that algorithm can be used for analyzing the output data.

In the first line, the number of bodies Nbg and index lengths ig and ig2 are defined. Here, instead of the operator READPRN, the operator READ is used. The latter operator permits reading-out of a non-ordered file. After the reading, required quantities can be displayed as shown in the graph.

Item 14.12.14 gives changes for files generation of problems on interaction of two, 175 and 473 particles.

#### **14.13. Problem solving procedure**

Section 13 gives recommendations about the problem solution sequence of gravitational interaction. They are applicable as well for Coulomb's interaction.

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**Appendix 1. Example of MathCad program for generating an initial condition file  
for gravitational interaction**

**InCnPrpr.mcd . Example of calculation of the initial conditions for the program Galactica. Completed 11.02.2012, modified 17.09.2012. 18:00**

12.1. Number of bodies

$$Nb := 30 \quad N := Nb - 1 \quad i := 0..N \quad i1 := 1..N$$

12.2. Constants

$$G := 6.67259 \cdot 10^{-11} \quad AU := 1.49597870691 \cdot 10^{11} \quad P_{sd} := 365.25636042 \quad k_{sd\_Jc} := \frac{P_{sd}}{36525}$$

$$dsc := 24 \cdot 3600 \quad P_{sdsc} := P_{sd} \cdot dsc$$

12.3. Properties of bodies and of their motions

$$mEr := 5.9742 \cdot 10^{24} \quad M_S := 332951.3 \cdot mEr \quad M_{SS} := 1.991787350282 \cdot 10^{30}$$

$$Roba := 5000 \quad k_a := 1.0 \quad a := k_a \cdot AU \quad e := 0.2056 \quad \epsilon_0 := 0.409$$

12.4. Masses of bodies in kg and their radiuses in m

$$m_0 := M_S \quad m_{i1} := \frac{M_{SS} - M_S}{N} \quad ram_1 := \left( \frac{3 \cdot m_1}{4 \cdot \pi \cdot Roba} \right)^{\frac{1}{3}}$$

**12.5. Coordinates and velocities of bodies accordingly to:**

Smulsky J.J. Axisymmetrical problem of gravitational interaction of N-bodies // Mathematical mode 2003, Vol. 15, No 5, Pp. 27-36. (In Russian <http://www.smul1.newmail.ru/Russian1/IntSunSyst/Osvnb4.doc>).

$$\alpha_1 := -\frac{1}{1+e} \quad Rp := a \cdot \frac{2 \cdot \alpha_1 + 1}{\alpha_1} \quad f_N := 0.25 \cdot \sum_{i2=2}^N \frac{1}{\sin \left[ \frac{(i2-1) \cdot \pi}{N} \right]} \quad \mu_1 := -G \cdot (m_0 + m_1 \cdot f_N)$$

$$vp := \sqrt{\frac{\mu_1}{\alpha_1 \cdot Rp}} \quad d\phi := \frac{2 \cdot \pi}{N} \quad \phi_{0_{i1}} := (i1 - 1) \cdot d\phi \quad \alpha_1 = -0.829462508294625$$

**Coordinates and velocities in the plane of orbits, in m and m/sec**

$$x_{0_{i1}} := Rp \cdot \cos(\phi_{0_{i1}}) \quad y_{0_{i1}} := Rp \cdot \sin(\phi_{0_{i1}}) \quad z_{0_{i1}} := 0 \quad \mu_1 = -1.32824690980362 \cdot 10^{20}$$

$$vx_{0_{i1}} := -vp \cdot \sin(\phi_{0_{i1}}) \quad vy_{0_{i1}} := vp \cdot \cos(\phi_{0_{i1}}) \quad vz_{0_{i1}} := 0 \quad f_N = 16.12083740509405$$

**Checking the distanses between bodies. If d is less than 50, it is need to increase a.**

$$do := \sqrt{(x_{0_2} - x_{0_1})^2 + (y_{0_2} - y_{0_1})^2 + (z_{0_2} - z_{0_1})^2} \quad d := \frac{do}{(2 \cdot ram_1)} \quad d = 784.3895396357486$$

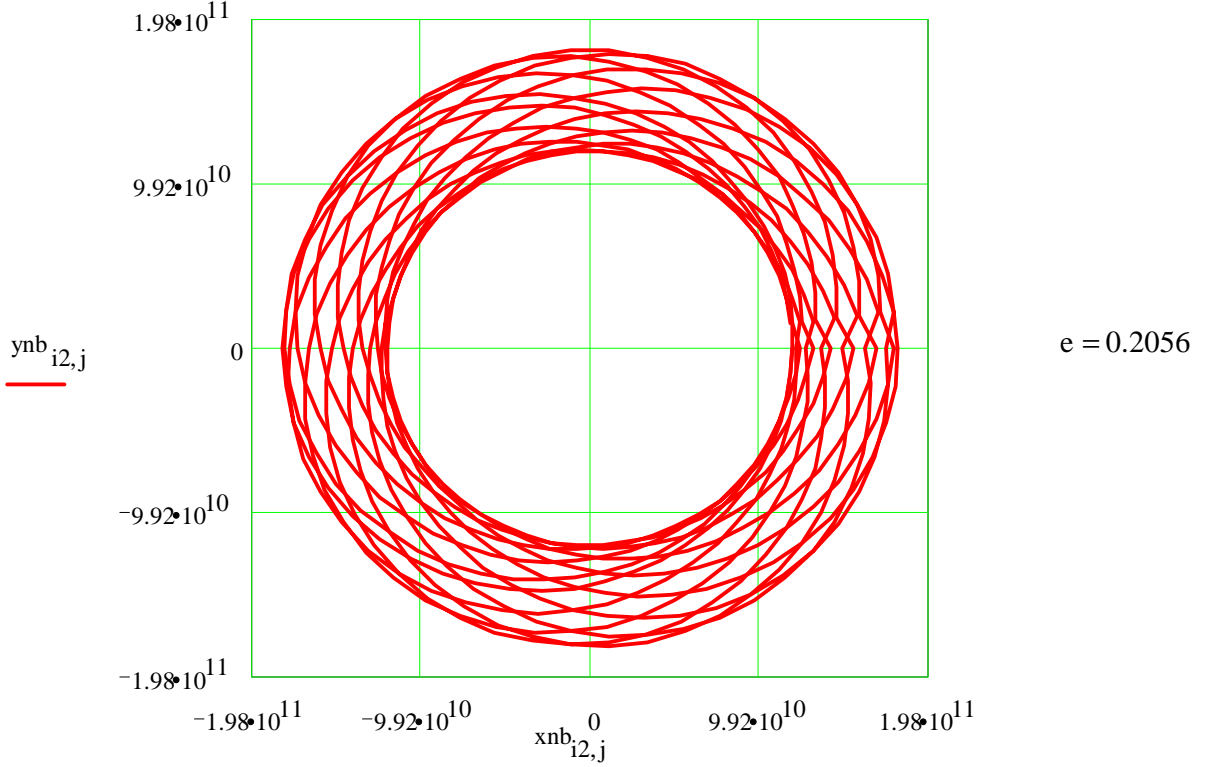
**Appendix 1 (continued)**

## 12.6. The exact motion of bodies. Checking the planned configuration of the problem

$$J := 50 \quad \phi_0 := 0 \quad j := 1..J \quad \phi_j := \phi_{j-1} + \frac{2 \cdot \pi}{J}$$

$$\text{rnb}_{11,j} := \frac{\text{Rp}}{\left[ (\alpha_1 + 1) \cdot \cos(\phi_j - \phi_{011}) - \alpha_1 \right]} \quad \text{Ra} := \frac{\text{Rp}}{\left[ -(\alpha_1 + 1) - \alpha_1 \right]} \quad \text{D}_M := 1.1 \cdot \text{Ra}$$

$$\text{xn}_{11,j} := \text{rnb}_{11,j} \cdot \cos(\phi_j) \quad \text{yn}_{11,j} := \text{rnb}_{11,j} \cdot \sin(\phi_j) \quad i2 := 1, 3..N$$



## 12.7. Coordinates and velocities in the equatorial plane

$$\text{xm}_{11} := \text{xo}_{11} \quad \text{ym}_{11} := \text{yo}_{11} \cdot \cos(\epsilon_0) - \text{zo}_{11} \cdot \sin(\epsilon_0) \quad \text{zm}_{11} := \text{yo}_{11} \cdot \sin(\epsilon_0) + \text{zo}_{11} \cdot \cos(\epsilon_0)$$

$$\text{vxms}_{11} := \text{vx}_{11} \quad \text{vym}_{11} := \text{vy}_{11} \cdot \cos(\epsilon_0) - \text{vz}_{11} \cdot \sin(\epsilon_0) \quad \text{vzms}_{11} := \text{vy}_{11} \cdot \sin(\epsilon_0) + \text{vz}_{11} \cdot \cos(\epsilon_0)$$

$$\text{xm}_0 := 0 \quad \text{ym}_0 := 0 \quad \text{zm}_0 := 0 \quad \text{vxms}_0 := 0 \quad \text{vym}_0 := 0 \quad \text{vzms}_0 := 0$$

## 12.8. The transition to the dimensionless variables

$$\text{ms}_0 := 0 \quad \text{ms}_{1+1} := \text{ms}_1 + m_1 \quad \text{mo}_1 := \frac{m_1}{\text{ms}_{N+1}}$$

**Determination of the length scale that the dimensionless time unit is equal one sidereal century**

$$k_t := \frac{1}{100 \cdot P_{\text{sdsc}}} \quad \text{Am} := \left[ \frac{(G \cdot \text{ms}_{N+1})^{\frac{1}{3}}}{k_t^2} \right] \quad \text{Am} = 1.097960770309581 \cdot 10^{13}$$

**Appendix 1 (continued)**

### Clarification of Am

$$Am := 1.097960770309582 \cdot 10^{13} \quad k_v := \sqrt{\frac{Am}{G \cdot ms_{N+1}}} \quad Am := \frac{1}{k_t \cdot k_v} \quad Am = 1.097960770309582 \cdot 10^{13}$$

$$x_i := \frac{xm_i}{Am} \quad y_i := \frac{ym_i}{Am} \quad z_i := \frac{zm_i}{Am} \quad ra_i := \frac{ram_i}{Am}$$

$$vx_i := vxm_i \cdot k_v \quad vy_i := vym_i \cdot k_v \quad vz_i := vzm_i \cdot k_v$$

### 12.9. The center of mass of the interacting bodies

$$Xc_0 := 0 \quad Yc_0 := 0 \quad Zc_0 := 0 \quad Vxc_0 := 0 \quad Vyc_0 := 0 \quad Vzc_0 := 0$$

$$Xc_{i+1} := Xc_i + mo_i \cdot x_i \quad Yc_{i+1} := Yc_i + mo_i \cdot y_i \quad Zc_{i+1} := Zc_i + mo_i \cdot z_i$$

$$Vxc_{i+1} := Vxc_i + mo_i \cdot vx_i \quad Vyc_{i+1} := Vyc_i + mo_i \cdot vy_i \quad Vzc_{i+1} := Vzc_i + mo_i \cdot vz_i$$

### Coordinates and velocities relatively the center of mass

$$x_i := x_i - Xc_{N+1} \quad y_i := y_i - Yc_{N+1} \quad z_i := z_i - Zc_{N+1} \quad Xc_{N+1} = -1.058791184067875 \cdot 10^{-22}$$

$$vx_i := vx_i - Vxc_{N+1} \quad vy_i := vy_i - Vyc_{N+1} \quad vz_i := vz_i - Vzc_{N+1} \quad Vxc_{N+1} = -3.388131789017201 \cdot 10^{-19}$$

### 12.10. Angular momentum of the system of bodies

$$Mx_i := 0 \quad My_i := 0 \quad Mz_i := 0$$

$$M1x_i := -mo_i \cdot (vy_i \cdot z_i - vz_i \cdot y_i) \quad M1y_i := mo_i \cdot (vx_i \cdot z_i - vz_i \cdot x_i) \quad M1z_i := mo_i \cdot (vy_i \cdot x_i - vx_i \cdot y_i)$$

$$Mx_{i+1} := (Mx_i) + M1x_i \quad My_{i+1} := (My_i) + M1y_i \quad Mz_{i+1} := (Mz_i) + M1z_i$$

$$Mx_{N+1} = 1.97659249147141 \cdot 10^{-23} \quad My_{N+1} = -6.087427365523565 \cdot 10^{-5}$$

$$Mz_{N+1} = 1.404436095090305 \cdot 10^{-4}$$

### Verification

$$msr_0 := 0 \quad msr_{i+1} := msr_i + mo_i \quad msr_{N+1} = 1.0000000000000002$$

$$M0 := \frac{N \cdot mo_i \cdot Rp \cdot vp \cdot k_v}{Am} \quad M0 := \sqrt{(Mx_{N+1})^2 + (My_{N+1})^2 + (Mz_{N+1})^2}$$

$$\frac{M0 - M00}{M0} = -1.770774882285912 \cdot 10^{-16} \quad \left( Am - \frac{1}{k_t \cdot k_v} \right) \cdot \frac{1}{Am} = 0$$

**12.11. To coordinate the size of area with the number of bodies need to change the B <= 1 and C1 <= 1 so that Nbar > = Nb.**

$$B := 0.5 \quad C1 := 0.25 \quad Mu := \text{ceil} \left[ \left( Nb + 1 \right)^{\frac{1}{3}} \right] \quad dT := 1 \cdot 10^{-6} \quad Mu = 4$$

$$Nbar := (1 + Mu) \cdot (1 + Mu \cdot B) \cdot (1 + Mu \cdot C1) \quad Nbar = 30 \quad Nb = 30$$

B	C	Mu	Nb
0.5	0.25	4	30
0.9	0.6	7	300
0.9	0.7955	15	3000



### 12.12. Writing data to the file name.prn

```

k := 0..14      l := 7..12      l2 := 5..14      R0,k := 0      R0,7 := MxN+1      R0,8 := MyN+1
R0,9 := MzN+1  R1,0 := 0      R1,1 := dT      R1,2 := 1      R1,3 := 1      R1,4 := Nb
R1,5 := 1      R1,6 := B      R1,7 := C1      R1,8 := Mu      Ri+2,0 := moi      Ri+2,1 := xi
Ri+2,2 := yi      Ri+2,3 := zi      Ri+2,4 := vxi      Ri+2,5 := vyi      Ri+2,6 := vzi      Ri+2,1 := 0
Ri+2,13 := rai      Ri+2,14 := 0

```

**Information line at end of file**    l3 := Nb + 2

```

R13,0 := 20120917  R13,1 := Nb  R13,2 := MSS  R13,3 := Am  R13,4 := kv  R13,12 := 0

```

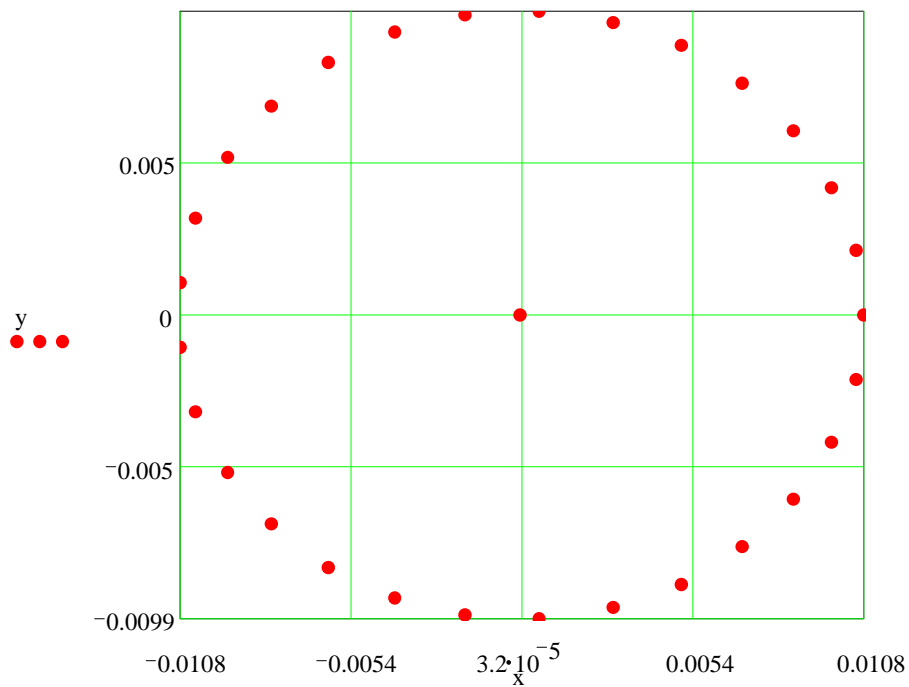
```

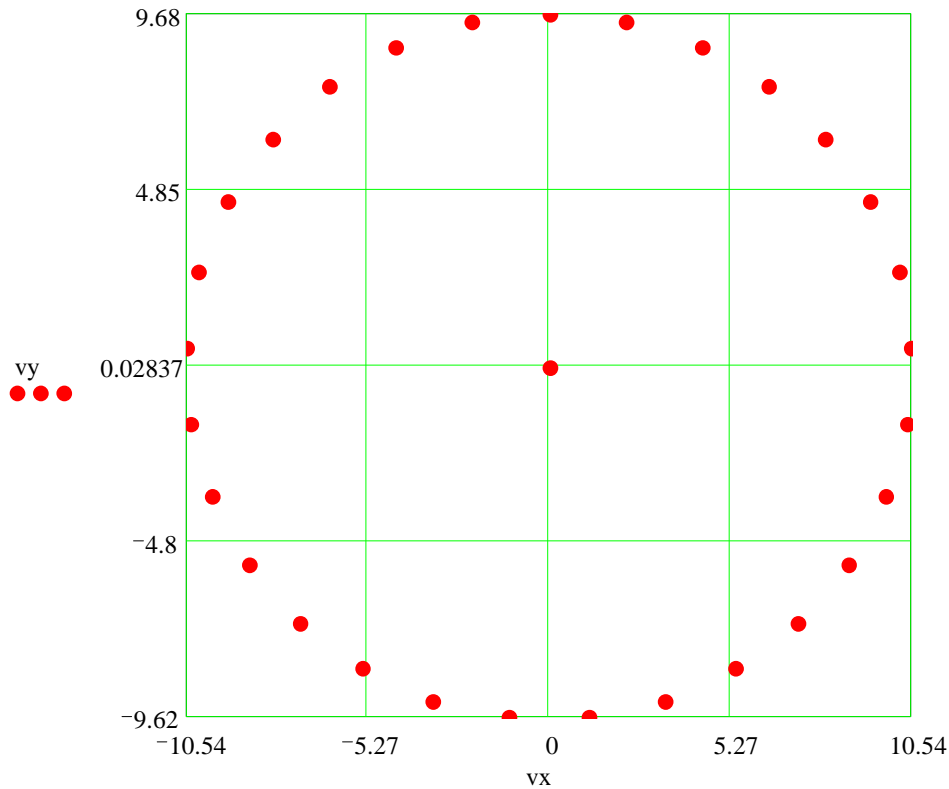
WwwwRITEPRN("axsymb3t.prn") := R      "axsymb30.prn"      "axsymb3c.prn"      "axsymb3t.prn"

```

**It is necessary 6 zero at the end of 2-nd line of a file name.prn to remove and to rename file into a name.dat.**

### Verification coordinates and velocities





**12.13. Calculating the number of steps and the step size for an accurate account from one d with Julian day JD1 to another date with the JD2.**

$$JD1949 := 2433280.5 \quad JD1969 := 2.4404005 \cdot 10^6 \quad JD1999 := 2.4515425 \cdot 10^6 \quad dT = 1 \cdot 10^{-6}$$

$$JD1 := JD1949 \quad JD2 := JD1969 \quad \Delta T_{jd} := \frac{JD2 - JD1}{36525} \quad \Delta T := \frac{\Delta T_{jd}}{k_{sd\_Jc}} \quad Ltkr := \frac{\Delta T}{dT}$$

$$Ltkr = 1.949315815284605 \cdot 10^7 \quad Ltk := 19500000$$

**At kl3 = 1 or = 3 for file garez28.dat**

$$dTc1 := \frac{Ltkr \cdot dT}{Ltk} \quad dTc1 = 9.996491360433874 \cdot 10^{-7}$$

**At kl3 = 2 or = 4 for file name.dat**

$$dTc2 := \frac{Ltkr \cdot dT}{Ltk - 0.999} \quad dTc2 = 9.996491872561842 \cdot 10^{-7}$$

**Verification**

$$Ltk \cdot dTc1 - \Delta T = 0$$

$$(Ltk - 1) \cdot dTc2 + 0.001 \cdot dTc2 - \Delta T = 0$$

**Appendix 1 (end)**

## 12.14. Reading the file of type garez28.dat

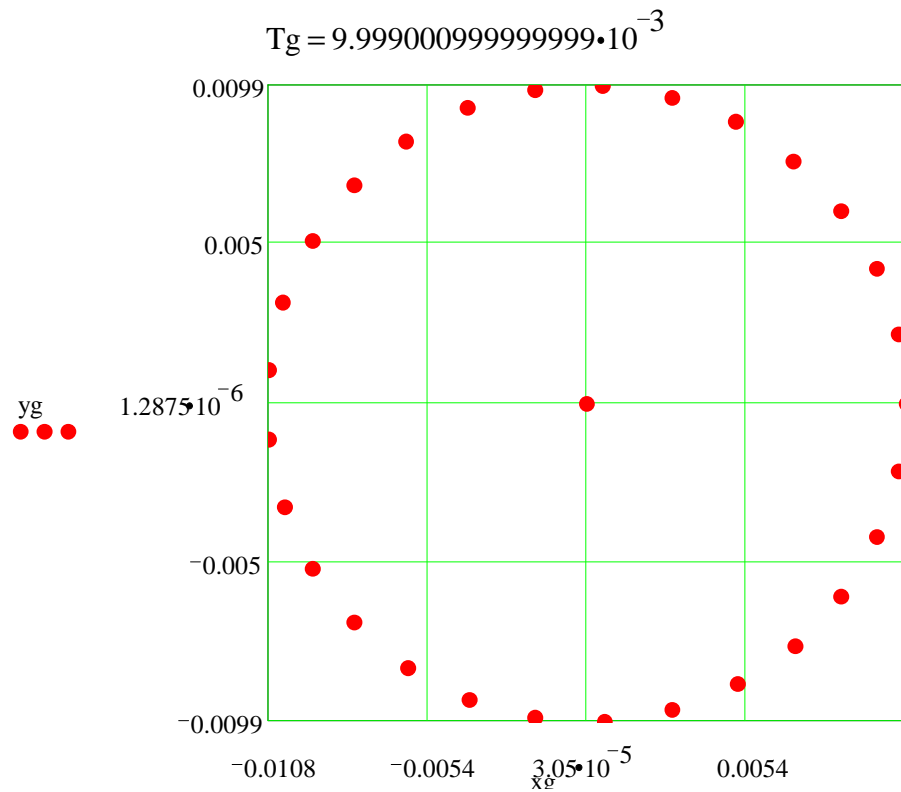
```

Nbg := 30   Ng := Nbg - 1   ig := 0..Ng   ig2 := 0..23 + Nbg·15
P2_ig2 := READ("garez28.dat")   "garez28.dat"   "g28c10"

Tg := P2_0   m_max := P2_1   v_max := P2_2   dTpg := P2_3   P_xg := P2_4   Pyg := P2_5   Pyg := P2_6
M_xg := P2_7   Myg := P2_8   M_zg := P2_9   S_sxg := P2_10   S_syg := P2_11   S_szg := P2_12
Eg := P2_13   E_g_max := P2_14   E_tg := P2_15   dTg := P2_16   i2b := P2_17   j2b := P2_18
k2b := P2_19   Ag := P2_20   Bg := P2_21   Cg := P2_22   Mug := P2_23

mog_ig := P2_24+ig·15   xg_ig := P2_25+ig·15   yg_ig := P2_26+ig·15   zg_ig := P2_27+ig·15
vxg_ig := P2_28+ig·15   vyg_ig := P2_29+ig·15   vzg_ig := P2_30+ig·15   rag_ig := P2_37+ig·15

```



Symbolical argument	Cod	Description
\$MAXRESMODE	-3	graphics mode with highest resolution
\$MAXCOLORMODE	-2	graphics mode with most colors
\$DEFAULTMODE	-1	restore screen to original mode
\$TEXTBW40	0	40 x 25 text, 16 grey
\$TEXTC40	1	40 x 25 text, 16/8 color
\$TEXTBW80	2	80 x 25 text, 16 grey
\$TEXTC80	3	80 x 25 text, 16/8 color
\$MRES4COLOR	4	320 x 200, 4 color
\$MRESNOCOLOR	5	320 x 200, 4 grey
\$HRESBW	6	640 x 200, BW
\$TEXTMONO	7	80 x 25 text, BW
\$HERCMONO	8	720 x 348, BW for HGC
\$MRES16COLOR	13	320 x 200, 16 color
\$HRES16COLOR	14	640 x 200, 16 color
\$ERESNOCOLOR	15	640 x 350, BW
\$ERESCOLOR	16	640 x 350, 4 or 16 color
\$VRES2COLOR	17	640 x 480, BW
\$VRES16COLOR	18	640 x 480, 16 color
\$MRES256COLOR	19	320 x 200, 256 color
\$ORESOLOR	64	640 x 400, 1 of 16 colors (Olivetti)

### Appendix 3. Example of MathCad program for generating an initial condition file

for Coulomb's interaction

**InCnPrCib.mcd . Example of calculation of the initial conditions for the program Galactica at Coulomb's interactions. Completed 03.09.2012, modified 17.09.2014.**

14.12.1. Number of bodies

$$Nb := 9 \quad N := Nb - 1 \quad i := 0..N \quad i1 := 1..N \quad N = 8$$

14. 12.2. Constants

From Handbook by Yavorsky & Detlaf: pp.749, 910, 912, 913.

$$m_e := 9.1091 \cdot 10^{-31} \quad m_p := 1.67252 \cdot 10^{-27} \quad m_{ne} := 1.67482 \cdot 10^{-27} \quad e_e := 4.80298 \cdot 10^{-10}$$

$$R0 := 1.5 \cdot 10^{-15} \quad R_e := 1.5 \cdot 10^{-15} \quad a_{B0} := 5.29167 \cdot 10^{-11} \quad \epsilon_d := 1 \quad a_k := 1$$

14.12.3. Properties of bodies and of their motions

$$A_N := 16 \quad Z_p := 8 \quad n_n := 2 \quad l_n := 1 \quad r_m := a_{B0} \cdot \frac{[3 \cdot n_n^2 - 1_n \cdot (1_n + 1)]}{2 \cdot Z_p} \quad r_n := R0 \cdot A_N^{\frac{1}{3}}$$

$$a := r_m \cdot a_k \quad e := 0.15 \quad \epsilon_0 := 0.409 \quad a = 3.30729375 \cdot 10^{-11} \quad r_n = 3.77976314968462 \cdot 10^{-15}$$

14.12.4. Masses of bodies in kg and their radiuses in m

$$Z_n := A_N - Z_p \quad m_0 := Z_p \cdot m_p + Z_n \cdot m_{ne} \quad m_{i1} := m_e \quad ram_0 := r_n \quad ram_{i1} := R_e$$

14.12.5. Coordinates and velocities of bodies accordingly to:

mulsky J.J. Axisymmetrical problem of gravitational interaction of N-bodies // Mathematical modelling. - 2003, Vol. 15, No 5, Pp. 27-36. (In Russian

<http://www.smul1.newmail.ru/Russian1/IntSunSyst/Osvnb4.doc>).

For two bodies interactions see item 14.12.14.

$$\alpha_1 := -\frac{1}{1+e} \quad Rp := a \cdot \frac{2 \cdot \alpha_1 + 1}{\alpha_1} \quad f_N := 0.25 \cdot \sum_{i2=2}^N \frac{1}{\sin\left[\frac{(i2-1) \cdot \pi}{N}\right]}$$

$$\mu1 := -\frac{10^{-9} \cdot e^2}{m_1 \cdot \epsilon_d} \cdot (N - f_N) \quad \mu1 = -1.315657523093202 \cdot 10^3$$

$$vp := \sqrt{\frac{\mu1}{\alpha_1 \cdot Rp}} \quad d\phi := \frac{2 \cdot \pi}{N} \quad \phi_{0_{i1}} := (i1 - 1) \cdot d\phi \quad \alpha_1 = -0.869565217391304$$

Coordinates and velocities in the plane of orbits, in m and m/sec

$$x_{0_{i1}} := Rp \cdot \cos(\phi_{0_{i1}}) \quad y_{0_{i1}} := Rp \cdot \sin(\phi_{0_{i1}}) \quad z_{0_{i1}} := 0 \quad \mu1 = -1.315657523093202 \cdot 10^3$$

$$vx_{i1} := -vp \cdot \sin(\phi_{0,i1}) \quad vy_{i1} := vp \cdot \cos(\phi_{0,i1}) \quad vzo_{i1} := 0 \quad f_N = 2.804865846209121$$

**Checking the distanses between bodies. If d is less than 50, it is need to increase a.**

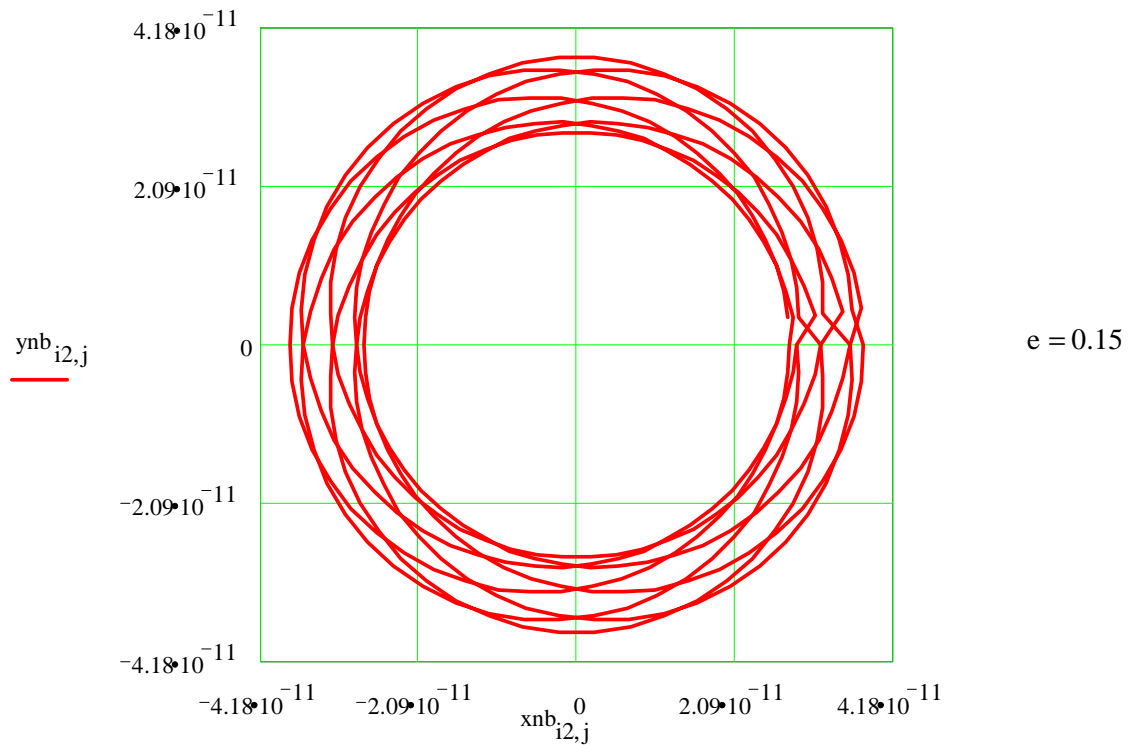
$$do := \sqrt{(xo_2 - xo_1)^2 + (yo_2 - yo_1)^2 + (zo_2 - zo_1)^2} \quad d := \frac{do}{(2 \cdot ram_1)} \quad d = 7.171996969841117 \cdot 10^3$$

#### 14.12.6. The exact motion of bodies. Checking the planned configuration of the problem

$$J := 50 \quad \phi_0 := 0 \quad j := 1..J \quad \phi_j := \phi_{j-1} + \frac{2 \cdot \pi}{J}$$

$$rmb_{i1,j} := \frac{Rp}{[(\alpha_1 + 1) \cdot \cos(\phi_j - \phi_{0,i1}) - \alpha_1]} \quad Ra := \frac{Rp}{[-(\alpha_1 + 1) - \alpha_1]} \quad D_M := 1.1 \cdot Ra$$

$$xnb_{i1,j} := rmb_{i1,j} \cdot \cos(\phi_j) \quad ynb_{i1,j} := rmb_{i1,j} \cdot \sin(\phi_j) \quad i2 := 1, 2..N$$



#### 14.12.7. Coordinates and velocities in the equatorial plane

$$xm_{i1} := xo_{i1} \quad ym_{i1} := yo_{i1} \cdot \cos(\epsilon_0) - zo_{i1} \cdot \sin(\epsilon_0) \quad zm_{i1} := yo_{i1} \cdot \sin(\epsilon_0) + zo_{i1} \cdot \cos(\epsilon_0)$$

$$vxms_{i1} := vx_{i1} \quad vyms_{i1} := vy_{i1} \cdot \cos(\epsilon_0) - vzo_{i1} \cdot \sin(\epsilon_0) \quad vzms_{i1} := vy_{i1} \cdot \sin(\epsilon_0) + vzo_{i1} \cdot \cos(\epsilon_0)$$

$$xm_0 := 0 \quad ym_0 := 0 \quad zm_0 := 0 \quad vxms_0 := 0 \quad vyms_0 := 0 \quad vzms_0 := 0$$

**Appendix 3 (continued)**

#### 14.12.8. The transition to the dimensionless variables

$$ms_0 := 0 \quad ms_{i+1} := ms_i + m_i \quad mo_i := \frac{m_i}{ms_{N+1}} \quad P_M := 1 \cdot 10^{-15}$$

$$qo_0 := Z_n \quad qo_{i1} := -1$$

**Determination of the length scale that the dimensionless time unit is equal one  $10^{-15}$  of second.**

$$k_t := \frac{1}{P_M} \quad Am := \left[ \frac{(10^{-9} \cdot e^2)}{\epsilon d \cdot ms_{N+1} \cdot k_t^2} \right]^{\frac{1}{3}} \quad Am = 2.049767130503777 \cdot 10^{-11}$$

#### Clarification of Am

$$Am := 2.049767130503774 \cdot 10^{-11} \quad k_v := \sqrt{\frac{\epsilon d \cdot ms_{N+1} \cdot Am}{10^{-9} \cdot e^2}} \quad Am := \frac{1}{k_t \cdot k_v}$$

$$x_i := \frac{xm_i}{Am} \quad y_i := \frac{ym_i}{Am} \quad z_i := \frac{zm_i}{Am} \quad ra_i := \frac{ram_i}{Am} \quad Am = 2.049767130503774 \cdot 10^{-11}$$

$$vx_i := vxm_i \cdot k_v \quad vy_i := vym_i \cdot k_v \quad vz_i := vzm_i \cdot k_v \quad \frac{2 \cdot a}{Am} = 3.226994618834738$$

#### 14.12.9. The center of mass of the interacting bodies

$$Xc_0 := 0 \quad Yc_0 := 0 \quad Zc_0 := 0 \quad Vxc_0 := 0 \quad Vyc_0 := 0 \quad Vzc_0 := 0$$

$$Xc_{i+1} := Xc_i + mo_i \cdot x_i \quad Yc_{i+1} := Yc_i + mo_i \cdot y_i \quad Zc_{i+1} := Zc_i + mo_i \cdot z_i$$

$$Vxc_{i+1} := Vxc_i + mo_i \cdot vx_i \quad Vyc_{i+1} := Vyc_i + mo_i \cdot vy_i \quad Vzc_{i+1} := Vzc_i + mo_i \cdot vz_i$$

#### Coordinates and velocities relatively the center of mass

$$x_i := x_i - Xc_{N+1} \quad y_i := y_i - Yc_{N+1} \quad z_i := z_i - Zc_{N+1} \quad Xc_{N+1} = -2.032879073410321 \cdot 10^{-20}$$

$$vx_i := vx_i - Vxc_{N+1} \quad vy_i := vy_i - Vyc_{N+1} \quad vz_i := vz_i - Vzc_{N+1} \quad Vxc_{N+1} = 5.204170427930421 \cdot 10^{-18}$$

#### 14.12.10. Angular momentum of the system of bodies

$$Mx_i := 0 \quad My_i := 0 \quad Mz_i := 0$$

$$M1x_i := -mo_i \cdot (vy_i \cdot z_i - vz_i \cdot y_i) \quad M1y_i := mo_i \cdot (vx_i \cdot z_i - vz_i \cdot x_i) \quad M1z_i := mo_i \cdot (vy_i \cdot x_i - vx_i \cdot y_i)$$

$$Mx_{i+1} := (Mx_i) + M1x_i \quad My_{i+1} := (My_i) + M1y_i \quad Mz_{i+1} := (Mz_i) + M1z_i$$

$$Mx_{N+1} = -9.665352161902836 \cdot 10^{-19} \quad My_{N+1} = -0.053108187925464$$

$$Mz_{N+1} = 0.122526400051667$$

#### Verification

$$msr_0 := 0 \quad msr_{i+1} := msr_i + mo_1 \quad msr_{N+1} = 1$$

$$M00 := \frac{N \cdot mo_1 \cdot Rp \cdot vp \cdot k_v}{Am} \quad M0 := \sqrt{(Mx_{N+1})^2 + (My_{N+1})^2 + (Mz_{N+1})^2}$$

$$\frac{M0 - M00}{M0} = 0 \quad \left( Am - \frac{1}{k_t \cdot k_v} \right) \cdot \frac{1}{Am} = 0$$

14.12.11. To coordinate the size of area with the number of bodies need to change the B <= 1 and C1 <= 1 so that Nbar >= Nb.

$$B := 0.25 \quad C1 := 0.1 \quad \text{Mu} := \text{ceil} \left[ (Nb + 1)^{\frac{1}{3}} \right] \quad dT := 1.0 \cdot 10^{-7} \quad \text{Mu} = 3$$

B	C	Mu	Nb
0.5	0.25	4	30
0.9	0.6	7	300
0.9	0.7955	15	3000

$$\text{Nbar} := (1 + \text{Mu}) \cdot (1 + \text{Mu} \cdot B) \cdot (1 + \text{Mu} \cdot C1) \quad \text{Nbar} = 9.1 \quad \text{Nb} = 9$$

#### 14.12.12. Writing data to the file name.prn

$$k := 0..14 \quad i := 7..12 \quad i2 := 5..14 \quad R_{0,k} := 0 \quad R_{0,7} := Mx_{N+1} \quad R_{0,8} := My_{N+1}$$

$$R_{0,9} := Mz_{N+1} \quad R_{1,0} := 0 \quad R_{1,1} := dT \quad R_{1,2} := 1 \quad R_{1,3} := 1 \quad R_{1,4} := Nb$$

$$R_{1,5} := 1 \quad R_{1,6} := B \quad R_{1,7} := C1 \quad R_{1,8} := \text{Mu} \quad R_{1,9} := \varepsilon d \quad R_{i+2,0} := mo_i \quad R_{i+2,1} := x_i$$

$$R_{i+2,2} := y_i \quad R_{i+2,3} := z_i \quad R_{i+2,4} := vx_i \quad R_{i+2,5} := vy_i \quad R_{i+2,6} := vz_i \quad R_{i+2,7} := 0$$

$$R_{i+2,13} := ra_i \quad R_{i+2,14} := 0 \quad R_{i+2,15} := qo_i$$

Information line at end of file  $i3 := Nb + 2$

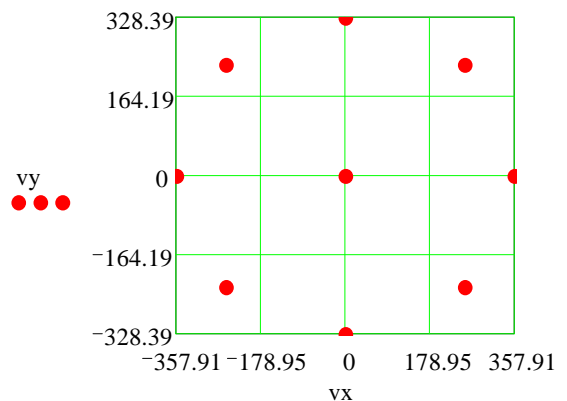
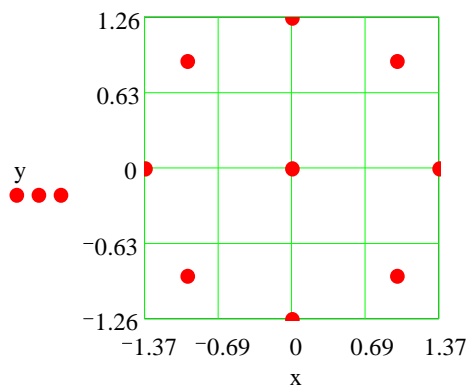
$$R_{i3,0} := 20120918 \quad R_{i3,1} := Nb \quad R_{i3,2} := ms_{N+1} \quad R_{i3,3} := Am \quad R_{i3,4} := k_v \quad R_{i3,12} := 0$$

$$\text{WwwwRITEPRN}("axsykl09.prn") := R \quad e \quad 0.15 \quad 0 \quad 0.15 \quad 0.15$$

"axsykl09.prn" "axsykc09.prn" "axsyk175.prn" "axsyk473.prn"

It is necessary 1 zero at the end of 1-nd line and 6 zero at the end of 2-nd line of file name.prn to remove and to rename the file into a name.dat.

#### Verification coordinates and velocities



Appendix 3 (end)



### 14.12.13. Reading the file of type garek29.dat

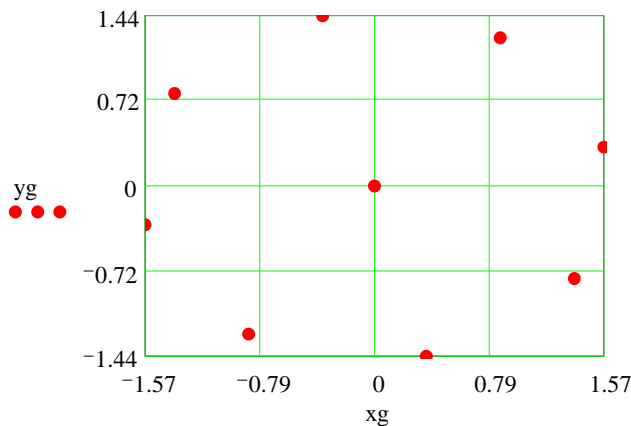
```

Nbgrd := 9      Ng := Nbgrd - 1      ig := 0..Ng      ig2 := 0..24 + (Nbgrd + 1)·16
P2_ig2 := READ("garek29.dat")      "garek29.dat"      "axsyk473.dat"      "axsyk109.dat"

Tg := P2_0      m_max := P2_1      v_max := P2_2      dTpg := P2_3      P_xg := P2_4      Pyg := P2_5      P_yg := P2_6
M_xg := P2_7      M_yg := P2_8      M_zg := P2_9      S_xg := P2_10      S_yg := P2_11      S_zg := P2_12
Eg := P2_13      E_g_max := P2_14      E_tg := P2_15      dTg := P2_16      i2b := P2_17      j2b := P2_18
k2b := P2_19      Ag := P2_20      Bg := P2_21      Cg := P2_22      Mug := P2_23      edg := P2_24
mog_ig := P2_25+ig·16      xg_ig := P2_26+ig·16      yg_ig := P2_27+ig·16      zg_ig := P2_28+ig·16
vxg_ig := P2_29+ig·16      vyg_ig := P2_30+ig·16      vzg_ig := P2_31+ig·16      duxg_ig := P2_32+ig·16      duyg_ig := P2_33+ig·16
duzg_ig := P2_34+ig·16      Sp_xg_ig := P2_35+ig·16      Sp_yg_ig := P2_36+ig·16      Sp_zg_ig := P2_37+ig·16
rag_ig := P2_38+ig·16      E_tg_ig := P2_39+ig·16      qog_ig := P2_40+ig·16      Datag := P2_25+Nbgrd·16

Nbgrd := P2_26+Nbgrd·16      M_SSg := P2_27+Nbgrd·16      Amg := P2_28+Nbgrd·16      k_vg := P2_29+Nbgrd·16

```



Tg = 0.0299999001

### 14.12.14. Change in items for Nb-paricles interactions

$$\text{Nb} = 2 \quad \alpha_1 := -\frac{1}{1+e} \quad \text{Rp} := a \cdot \frac{2 \cdot \alpha_1 + 1}{\alpha_1} \quad m_{pr} := \frac{m_0 \cdot m_1}{m_0 + m_1} \quad \mu_1 := -\left( \frac{10^{-9} \cdot e \cdot e^2}{m_{pr} \cdot \epsilon d} \right)$$

Nb = 175    AN = 348    Zp = 174    nn = 174    PM = 2\*10<sup>-10</sup>    B = 0.8    C1 = 0.55

Nb = 473    AN = 946    Zp = 472    nn = 472    PM = 2\*10<sup>-9</sup>    B = 0.9    C1 = 0.676