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Change of Angular Momentum in the Dynamics of the Solar System

I. I. Smul'skii and O. I. Krotov
Earth Cryosphere Institute, Siberian Branch, Russian Academy of Sciences, Tyumen, Russia
e-mail: JSmulsky@mail.ru, kolleg@rambler.ru
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Abstract—It is quite often that the calculated trajectories of celestial bodies and spacecraft differ from their actual motion. This difference may be due to deficiencies of the methods used to calculate orbits and trajectories. Three such methods—the Galactica software, the DE406 ephemeris, and the Horizons systems—are tested using the change of angular momentum in motion calculations. The smallest change is obtained using Galactica, and the largest, using Horizons. The dynamics is studied of the angular momentum of the planets. The results can be used to control and improve motion calculation methods.

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1. INTRODUCTION

The results of the space studies of recent decades provide evidence that the calculated orbits of celestial bodies and trajectories of spacecraft may often be inconsistent with their observed motions. Similar evidence is provided by studies of the Solar System's evolution over geological time periods. This evidence has led some researchers to conclude that the motions of objects in the Solar System are generally chaotic, suggesting a likelihood of a future collapse of the Solar System [1], chaotic motions of asteroids after planetary encounters [2], etc. Other researchers address these inconsistencies by introducing, in addition to the Newtonian force of gravity, other, weaker influences such as the Yarkovsky effect [3], dark matter [4], radiation pressure, etc.

However, the indeterminacy of motion and the unclear nature of the forces contradict the spirit of mechanics. Apparently, before accepting the above changes, researchers need to check, within the framework of mechanics, the reliability of the existing methods for calculating motions. One measure for whether the solution of a mechanics problem is accurate is the observance of the laws of conservation. In this paper we investigate the conservation of angular momentum for the entire system of interacting bodies in calculating the dynamics of the Solar System by different methods.

2. CHANGE OF ANGULAR MOMENTUM IN THE GALACTICA SOFTWARE

Studying the motion of Apophis for different initial conditions and by different methods, we found [5–8] that the uncertainty of the asteroid's motion after approaching the Earth can be reduced by increasing methodological accuracy. Thus, we investigated the change of the angular momentum of the Solar System in numerical calculations of its motion by two methods. The first method is traditional. It is based on the standard dynamic model (SDM) and is implemented in programs for calculating the DE series ephemeris, in particular the DE406 [9], and in the Horizons system [10]. The second method is implemented in the Galactica software [11]. It is based on the Newtonian interaction of point masses, and differential equations of motion are integrated using a new high-precision method. Information on the problems solved using the Galactica software can be obtained from: http://www.ikz.ru/~smulski/Papers/Galct11R.pdf. The Galactica system, with the functionalities necessary to solve problems, is freely available at http://www.ikz.ru/~smulski/GalactcW/. Its description is provided in the following files: GalDiscrp.pdf (Russian) and GalDiscrpE.pdf (English).

One important indicator for measuring the reliability of a solution of a differential motion equation is the relative change of the system's angular momentum. In the absence of external influences on the system of interacting material points, the angular momentum of its motion e.g., in projection on the axis z, remains unchanged:

$$M_z = \sum_{i=1}^{n} m_i(v_{yi}\,x_i - v_{xi}y_i) = \text{const}, \qquad (1)$$

where m_i, x_i and y_i, and v_{xi} and v_{yi} are the mass, coordinates, and velocities of the ith body and n is the number of bodies in the system.

Therefore, the relative change of the momentum:

$$\delta M_z = (M_z - M_{z0})/M_{z0}, \qquad (2)$$

where M_{z0} is the angular momentum at a certain point in time, must be zero; i.e., $\delta M_z = 0$. If it is not zero, we

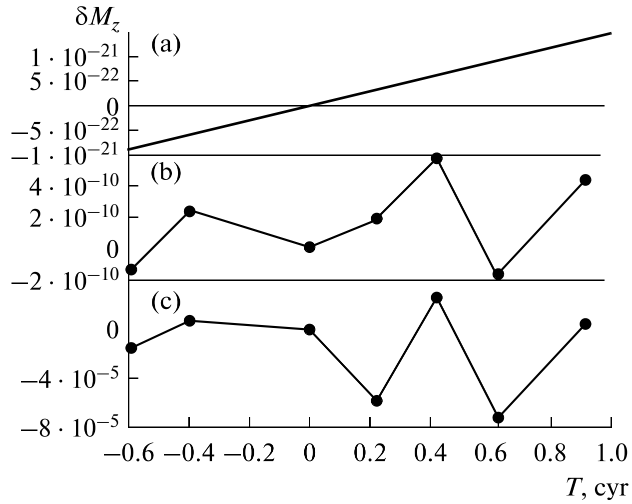

Fig. 1. Relative change of the angular momentum of the Solar System: (a) differential equations of motion of the Sun, planets, Moon, and Aphophis were integrated in Galactica; motion of the planets, Sun, Moon, and the three asteroids (Ceres, Pallas, and Vesta) were calculated using, (b) DE406, and (c) Horizons. The values of δM_z were calculated from (2) at M_{z0} as of November 30, 2008. T is time in Julian centuries of 36525 days in a century, from the epoch of November 30, 2008.

have evidence of errors in the numerical integration of the problem.

The measure of the accuracy of δM_z in the integration of equations in Galactica and the relationship of δM_z with the errors in the coordinates and velocities are detailed in [12, 13]. While solving differential equations, Galactica calculates various reliability criteria for the computed results, including the relative change of the momentum, δM_z. Repeated studies for the Solar System have found that the projections of angular momentum onto the axes x and y behave similarly to the projection of δM_z. Since this projection is close in value to a change in the modulus of the momentum, δM_t, below we consider only δM_z.

The angular momentum was calculated in Galactica for the planets, Moon, Sun, and Apophis in barycentric equatorial coordinates for the epoch 2000.0 [6–8]. The calculations were made with a step of $dT = 10^{-5}$ year and an extended length of numbers (34 decimal places). The pattern of change of δM_z over 160 years is shown in Fig. 1a. It is evident that this value changes linearly with time at an average rate of $d\delta M_z/dT = 1.5 \cdot 10^{-21}$ cyr^{-1}, where 1 cyr = 100 yrs. As already mentioned above, these results were obtained using numbers of extended length. When integrating the equation of motion using Galactica software with the double length of numbers (17 decimal places) over this time interval, the error of the momentum δM_z varies in the range $\delta M_z = \pm 10^{-13}$, i.e., does not increas linearly with the increase in the time needed to solve the problem. The algorithm of the Galactica software allows for error stabilization (if necessary) also in the case of the extended number length.

3. CHANGE OF ANGULAR MOMENTUM IN THE SDM SOFTWARE

We studied the change of angular momentum using the DE406 ephemeris and the Horizons system for the planets, the Sun, the Moon, and three asteroids—Ceres, Pallas, and Vesta—relative to the center of mass of the Solar System. We calculated the projections of the momentum M_x, M_y, and M_z onto the axes of the barycentric equatorial frame and momentum modulus M_t. All the calculations were performed for several time points. The body masses for the DE406 ephemeris (the same as in the DE405 ephemeris) were taken from their description.

The Horizons system also assigns a mass to each body. Since these masses differ from those used in the DE406 ephemeris, we also calculated the angular momenta with the masses from the DE406 ephemeris. Moreover, Horizons has Pluto's coordinates until January 29, 2051. Therefore, we calculated the angular momenta without Pluto. However, it turned out that the pattern of change of the angular momenta in the two latter cases is virtually the same as in the first case. Thus, in our further work we used the angular momenta with the masses from the DE405 ephemeris.

Table 1 presents the momenta M_z calculated using the DE406 ephemeris and Horizons for a period of 160 years. For the DE406 ephemeris, the values of the momentum are unchanged to the 10th significant digit; in the Horizons system, to their 4th significant digit. The pattern of change for the projections of the momentum M_x and M_y and the total momentum M_t is similar to the change in the z-projection of the momentum M_z; therefore, in what follows, we consider, like in Galactica, only the projection of the momentum onto the z axis.

Figure 1 compares the relative changes in the angular momenta calculated using Galactica, DE406, and Horizons. The changes in the momenta are given with respect to the momentum as of November 30, 2008. The first point corresponds to December 30, 1949. As already noted, in Galactica the angular momentum grows linearly with time, and its relative change over 160 years was $\delta M_z = 2.4 \cdot 10^{-21}$. In the DE406 ephemeris, δM_z changes nonmonotonically, and the range of the variations is $8 \cdot 10^{-10}$, which is 11 orders of magnitude greater than the momentum in Galactica.

The angular momentum in Horizons also changes nonmonotonically, and the variations in δM_z can be as large as $9 \cdot 10^{-5}$. Hence it follows that, first, the changes in the angular momentum in the DE406 ephemeris and in Horizons are many orders of magnitude greater than those in Galactica. Second, the changes in the angular momentum in Horizons are five orders of magnitude greater than those in the DE406 ephemeris.

Table 1. Angular momentum M_z of the motion of the planets, the Sun, the Moon, and three asteroids, which was calculated using the DE406 ephemeris and the Horizons system for different dates and numbers of Julian days (JD) with the masses from DE405

Date	JD	M_z, $\times 10^{+43}$ kg m^2/s	
		DE406	Horizons
Dec. 30, 1949	2433280.5	2.884103707433978	2.884087593847136
June 28, 1969	2440400.5	2.884103708561933	2.884148971531926
Nov. 30, 2008	2454800.5	2.884103707836915	2.884131506700124
Nov. 30, 2030	2462835.5	2.884103708363054	2.883964569598089
Nov. 30, 2050	2470140.5	2.884103709521903	2.884202731605625
Nov. 30, 2070	2477445.5	2.88410370733108	2.883923748548167
Nov. 30, 2099	2488037.5	2.884103709125478	2.884144694607399

It should be noted that originally the studies based on the DE406 ephemeris and the Horizons system were conducted for the planets, Moon, and Sun, i.e., without the three asteroids. The change in the momentum δM_z for the DE406 ephemeris was greater by a factor of 2.5. The results in Table 1 and Fig. 1 show a smaller change of δM_z because the DE406 based calculations took into account the three asteroids. Since the contribution of the asteroids to the change of the momentum δM_z is roughly $1.2 \cdot 10^{-9}$, it was expected that the consideration of the asteroids would not affect the change of the momentum in Horizons. This conclusion was confirmed by the calculations: the consideration of the asteroids did not affect the error of the angular momentum obtained using the Horizons system.

4. DYNAMICS OF THE ANGULAR MOMENTA OF INDIVIDUAL BODIES

To understand the reasons for the change of angular momentum, we studied these changes using the DE406 ephemeris for individual bodies: the planets, Sun, and Moon. We considered the relative change compared with the momentum as of November 30, 2008. We studied all the three projections of the momentum: δM_x, δM_y, and δM_z. Since their behavior is identical, we considered, like in the above, only the projection onto the z axis. The change δM_z for these bodies over 160 years is shown in Fig. 2 with a solid line. It is clear that the angular momenta of the bodies, like those of the Solar System in Fig. 1b, show oscillatory changes. The least relative changes are observed for Pluto, Neptune, Saturn, and Jupiter. The Sun's momentum shows the greatest change, and among the planets the greatest change δM_z is observed for Mercury.

It should be kept in mind that, unlike in the two-body problem, the interaction of more than two bodies results in a change in the angular momentum of each body. There is an ongoing exchange of momenta between the bodies. For example, it follows from the plots in Fig. 2 that the values of δM_z for Jupiter (Jp) and the Sun (Su) change asynchronously, which is evidence of an exchange of angular momenta between these bodies. Thus, the problem is not that these momenta change, but how correctly the results of the integration reflect the actual changes in the bodies' angular momenta. A slight inconsistency between the calculated and actual values may lead, due to their summation, to a visible change in the angular momentum for the Solar System as a whole.

The contribution of the angular momenta of individual bodies to that of the Solar System depends on their absolute values. Table 2 shows the momenta M_{z0i} of the bodies, the range $\Delta\delta M_{zi}$ of their relative changes, and the range of the absolute changes ΔM_{zi}. These values were found from the formula:

$$\Delta\delta M_{zi} = \delta M_{z\max i} - \delta M_{z\min i};\ \Delta M_{zi} = M_{z0i} \cdot \Delta\delta M_{zi},$$

where i is the number of the body and $\delta M_{z\max i}$ and $\delta M_{z\min i}$ are the maximum and minimum value of δM_{zi} in the plots in Fig. 2.

It is evident that the largest absolute range ΔM_{zi} of the variations in the angular momentum is observed for the Sun and Jupiter, and, as we see from Table 2, their ΔM_{zi} are similar. As noted above, their momenta change in antiphase. Therefore, the errors in the determination of their angular momenta may contribute substantially to δM_z of the Solar System as a whole.

The same studies of the angular momenta were conducted using the Galactica software. The relative changes in the momenta δM_z for the same bodies are shown in Fig. 2 by the dashed line. Here the calculations were conducted with a smaller time interval, i.e., every five years. For planets with a large rotation period, beginning with Jupiter, the angular momentum is seen to change periodically. For the terrestrial planets, the variation periods δM_z are less than the five-year interval between the points in the plots. Therefore, one cannot see the variations of these periods.

When comparing the relative momenta δM_z in the plots in Fig. 2, which were calculated using the DE406 ephemeris and Galactica, it is evident that their relative variation ranges are the same. In some cases, when

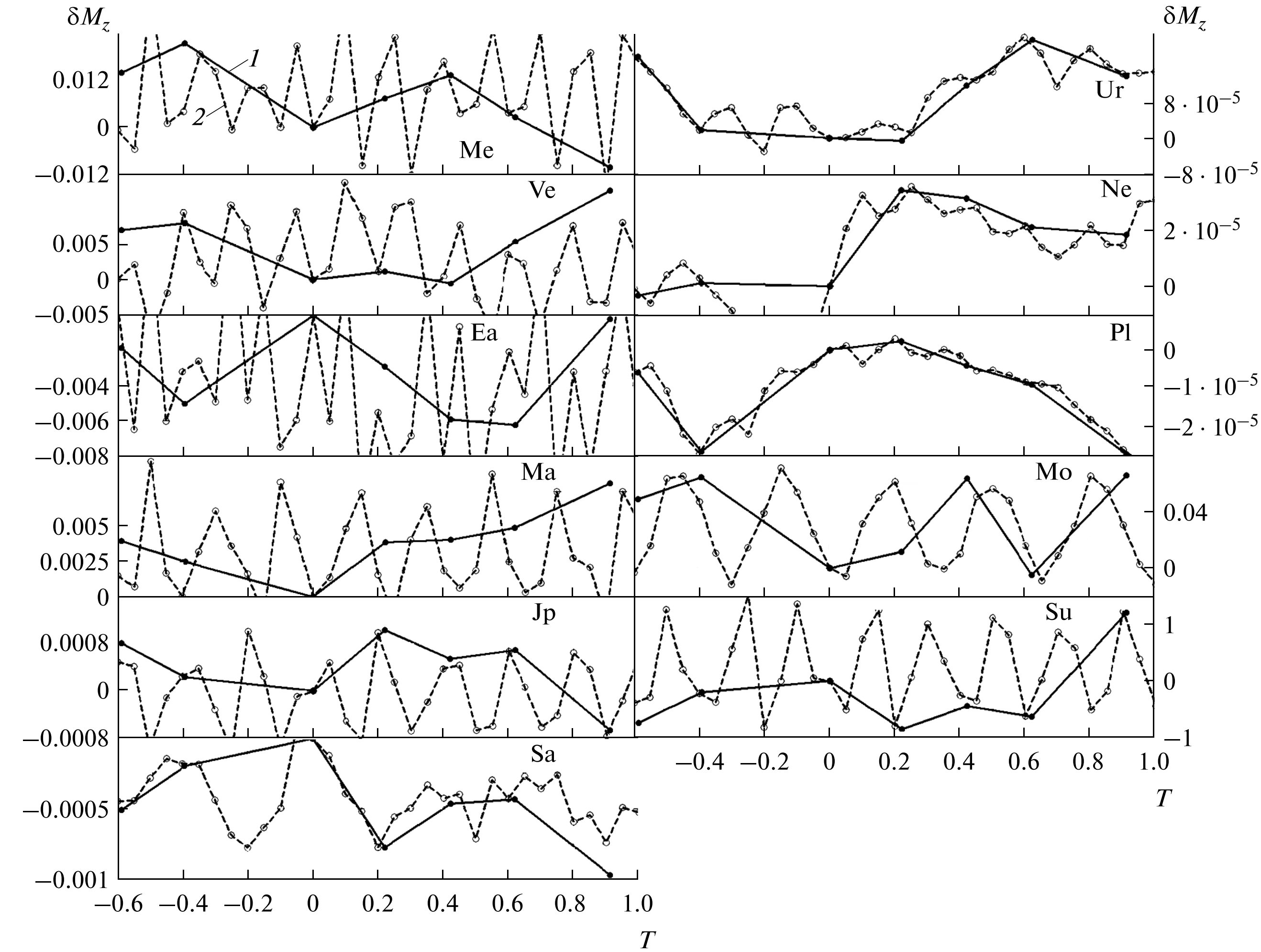

Fig. 2. Relative change of angular momenta for Solar System bodies from Mercury to the Moon and Sun. The value of δM_z was calculated from (2) at M_{z0} as of November 30, 2008: (*1*) using the DE406 ephemeris and (*2*) using Galactica.

the momenta are calculated for one and the same time point, the values of δM_z are also the same. For example, at $T \approx 0.4$ the relative changes in the momentum have approximately the same values for the following bodies: Me, Ve, Ea, Jp, Sa, Ur, Ne, Pl, and Su. It is only for two bodies—Mars (Ma) and the Moon (Mo)—that they are visibly different. As is evident from Fig. 1b, this difference for the DE406 ephemeris at $T \approx 0.4$ may lead to the largest error in the angular momentum for the whole Solar System: $\delta M_z = 6 \cdot 10^{-10}$.

A good consistency in the changes of the momenta δM_z for the two programs over the entire range is observed for Uranus (Ur), Neptune (Ne), and the Sun (Su). At the same time, the momenta δM_z are observed to differ at around certain points in time: $T = -0.6$ and -0.4 for Mercury, $T = -0.6$ and 0.9 for Venus, $T = 0.9$ for Saturn, and $T = 0.2$ and 0.6 for the Earth and Mars. These differences in the angular momenta for individual bodies may lead to the previously observed variations in the angular momentum for the whole Solar System in the DE406 ephemeris. Thus, the comparisons of the angular momenta for individual bodies (Fig. 2) by different methods can serve as landmarks in searching for the reasons for errors in the less accurate program, i.e., the DE406 ephemeris.

We used Galactica to study the change of angular momentum for the nearer planets with more detail. The periodicity in the change of coordinates and velocities, which, according to (1), determine the angular momentum, is due to the periodicity in the rotation of the bodies. Since the period P of the rotation of the bodies changes a thousandfold from Mercury to Pluto, the studies were conducted at time intervals divisible by the period P. Figure 3 shows its change δM_z during one rotation of the planet. As is evident from the plots, the value δM_z for all the planets in this interval undergoes oscillatory changes with periods less that P (planet's rotation). For the Earth (Ea), there are about 12 variations of δM_z, which are due to the lunar influence. The least variation range $\Delta\delta M_z \approx 3 \times 10^{-6}$ over the interval of one rotation is observed for Mercury, and the largest (if we ignore the Earth), for

Table 2. Ranges of change of angular momentum for the planets, Moon, and Sun relative to Solar System center of mass using DE406 for a period of 160 years from December 30, 1949. The relative changes were determined with respect to November 30, 2008. The projections of the bodies' angular momenta M_{z0i} and their changes ΔM_{zi} are given in kg m/s

Bodies	1	2	3	4	5	6
body	Me	Ve	Ea	Ma	Jp	Sa
$\Delta\delta M_{zi}$	0.0318	0.0132	0.00626	0.008	0.00172	0.000975
M_z	$7.795378332 \times 10^{38}$	1.6744633×10^{40}	2.4522183×10^{40}	3.1839633×10^{39}	1.7690015×10^{43}	7.2208333×10^{42}
ΔM_{zi}	2.4789303×10^{37}	2.2076076×10^{38}	1.5344546×10^{38}	$2.56960663 \times 10^{37}$	3.0355753×10^{40}	7.0420275×10^{39}
Bodies	7	8	9	10	11	
body	Ur	Ne	Pl	Mo	Su	
$\Delta\delta M_{zi}$	0.00231	0.000375	0.0000322	0.071	2.075	
M_z	1.551594×10^{42}	2.3175955×10^{42}	3.6622486×10^{38}	2.9202579×10^{38}	1.5101363×10^{40}	
ΔM_{zi}	3.5870122×10^{38}	8.6886268×10^{37}	1.1792440×10^{34}	2.0741525×10^{37}	3.1328189×10^{40}	

Jupiter: $\Delta\delta M_z \approx 2 \cdot 10^{-4}$. Due to the lunar influence, the value $\Delta\delta M_z = 10^{-3}$ for the Earth is greater than for Jupiter. Comparing these variation ranges with $\Delta\delta M_{zi}$ over 160 years (Table 2), we can see that the value of the variations during one rotation of the body is 3–4 orders of magnitude less for Mercury and Venus and one order of magnitude less for the Earth and Jupiter.

It should be noted that the dynamics of the angular momentum δM_z during one rotation (Fig. 3) can be different in a different epoch. Thus, we studied the changes in angular momentum over large time periods. We considered the average moduli of the angular momenta δM_{tm} during one rotation. These studies were carried out for each planet over an interval of 300 planetary rotations. Figure 4 shows the changes in the average angular momenta for the same planets as in Fig. 3. Since the interval between points in the plots in Fig. 4 is one planetary rotation period P, the variation periods for the angular momentum are equal to several periods P. For example, the least variation periods for the average angular momentum in Fig. 4 for Mercury and Jupiter are 4–5 of their rotation periods P. As is seen from Fig. 4, in addition to these short variations, there are also longer ones. And for Mars and Jupiter one can see tendencies that mark the beginning of variations with a period of tens or hundreds of thousands of years. They are due to the long-period variations of the planetary orbits [13].

The range of variations of the average angular momenta in Fig. 4 does not exceed that of variations during one rotation, which are shown in Fig. 3. The reason is that the averaging of the variation amplitudes during one rotation reduces their range. It should be noted that Figs. 3 and 4 show the studies on the changes in the momenta over large time intervals for the first five planets only. For the other planets, these changes are well seen in the plots (dashed line) in Fig. 2 with a time interval of 5 years. Over the 160-year interval for planets from Saturn (Sa) to Pluto (Pl), one can trace variations with two periods of the order of ten and of the order of one hundred years.

It should be noted that the high time resolution studies on the angular momentum M_z using the Galactica software (Fig. 3) show that the change δM_z for individual bodies is smooth, i.e., without any jumps or breaks. Therefore, the difference between δM_z calculated using DE406 (Fig. 2) and those calculated using Galactica is due to inaccuracies in the DE406 ephemeris.

Thus, despite the various changes in the angular momenta of the individual bodies of the Solar System, the angular momentum of the whole system remains unchanged. The degree of change indicates the accuracy of the solution of equations describing Solar System dynamics. Galactica gives the smallest change in angular momentum, and the Horizons system gives the greatest. The change of the angular momentum of individual bodies in the best calculation program can serve as a benchmark to determine the causes of errors in the less accurate calculation programs.

5. DIFFERENCES IN THE POSITIONS OF BODIES

The calculated changes of the angular momentum may indicate errors in the coordinates and velocities of bodies. We now try to estimate them. Let all the bodies have the same relative deviation δ for all coordinates and velocity components; then we can write the coordinate and velocity of the ith body, i.e., for the projection onto the x axis, at any point in time:

$$x_i = x_{ti}(1 + \delta); \; v_{xi} = v_{xti}(1 + \delta), \quad (3)$$

where x_i and v_{xi} are the calculated values and x_{ti} and v_{xti} are the true values of the coordinate and velocity of the ith body at this time point. If we substitute, according to (3), the coordinates and velocities into relation (1)

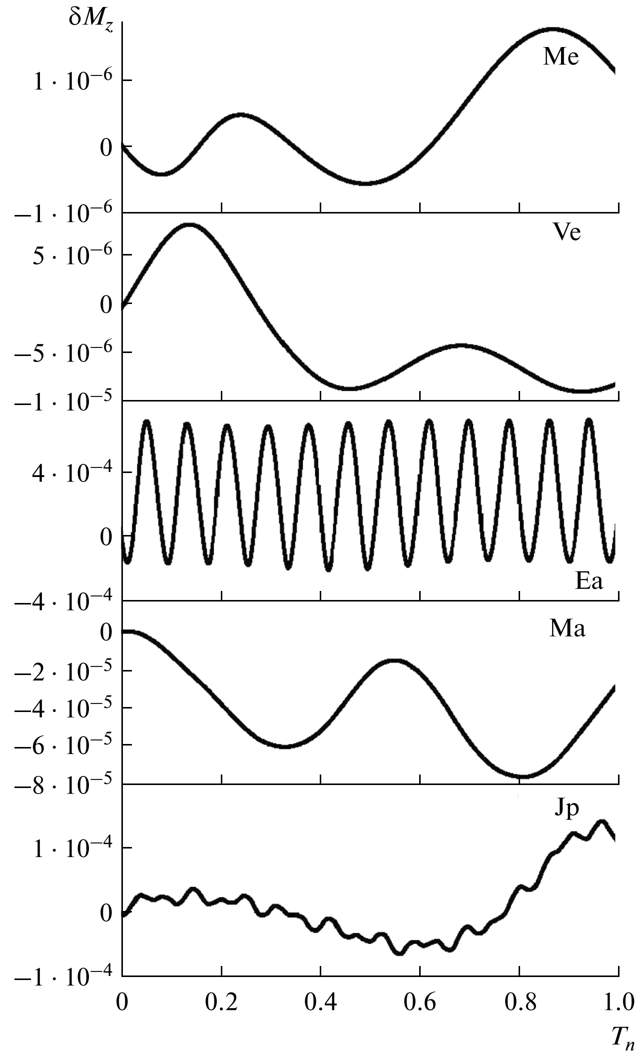

Fig. 3. Relative change of z-projection of angular momentum for planets from Mercury to Jupiter over one planetary rotation. The values of δM_z were calculated at M_{z0} as of November 30, 1949. $T_n = T/P$ is the normalized time in rotation periods. $P = 0.24$; 0.68; 1.000; 1.84; and 11.85 are rotation periods (in sidereal years) for planets from Mercury to Jupiter.

for angular momentum and then into (2), we obtain $\delta M_z \approx 2\delta$. It should be noted that in this case the calculation of the relative change in the momentum δM_z is based on M_{z0} in (2), which is calculated from the true values of x_{ti} and v_{xti}, etc.

Thus, given that the relative deviation of the coordinates and velocities is the same, it is half of the deviation of the momentum $\delta = 0.5\delta M_z$.

To analyze the structure of the deviations, we studied the differences between the DE406 ephemeris and the DE405, DE403, and DE200 ephemeris and the Horizons system for two dates: December 30, 1949 with the Julian day JD = 2433280.5 and December 30, 1999 with JD = 2451542.5. We determined the deviations of the coordinates Δx_i, Δy_i, and Δz_i and the velocities Δv_{xi}, Δv_{yi}, and Δv_{zi}; the deviations of the moduli of the distances Δr_i and velocities Δv_i; and the angular displacement $\Delta\varphi_i$ in the plane xy and the relative change in the distances between the positions of the body δr_i.

Table 3 gives two parameters of these studies, which were obtained by averaging over all bodies: δr_m is the average relative deviation of the distance between the bodies in different calculation programs and $\Delta\varphi_m$ is the average moduli of the difference of the angular distances between the bodies in the heliocentric equatorial frame. As is evident from Table 3, these values are well-correlated between each other, with $\Delta\varphi_m$ being approximately twice as small as δr_m. A comparison of two different epochs—1949 and 1999—shows that the pattern of the deviations is almost unchanged.

It is seen from Table 3 that the lower the number of an ephemeris, the worse is its accuracy. The data of Table 3 also confirm that the accuracy of the Horizons system is worse than that of the DE406 or DE405 ephemeris. Moreover, it follows from the analysis of the differences in the distances Δr and velocities Δv that although their values vary in a broad range for different bodies, their relative values δr and δv vary within narrower limits. The average value of the limits is accurately reflected by the values δr_m and $\Delta\varphi_m$. Therefore, the use of the same value for the deviation δ of the bodies' coordinates and velocities when deriving its dependence on the deviation δM_z of the angular momentum is justified.

When studying the changes in the angular momentum over 160 years, we found that the range of its variations is $\Delta\delta M_z = 8 \cdot 10^{-10}$ for the ephemeris and $9 \cdot 10^{-5}$ for Horizons. Therefore, the relative errors of the coordinates and velocities calculated using these systems should be expected to be of the order of $4 \cdot 10^{-10}$ and $4.5 \cdot 10^{-5}$, respectively. This accuracy estimate was obtained for the "true" parameters of the motion of the bodies, which give a constant angular momentum δM_z. Naturally, this estimate differs from the deviations δr_m in Table 3, which were obtained by comparing the different versions of the ephemeris.

6. CHANGE OF ANGULAR MOMENTUM WITH REGARD TO THE ROTATIONAL MOTION OF BODIES

In the foregoing we considered the total angular momentum of bodies in the dynamics of the Solar System, which is induced by their orbital motion. When analyzing our paper, the reviewer noted that the consideration of the angular momentum induced by the rotational motion of bodies would expand the possibilities of this approach. For example, in the Earth–Moon system, one could trace an increase in the orbital angular momentum of the Moon due to the inhibition of the Earth's rotation. Indeed, it is of interest to consider the total angular momentum, taking into account the angular momenta induced by the

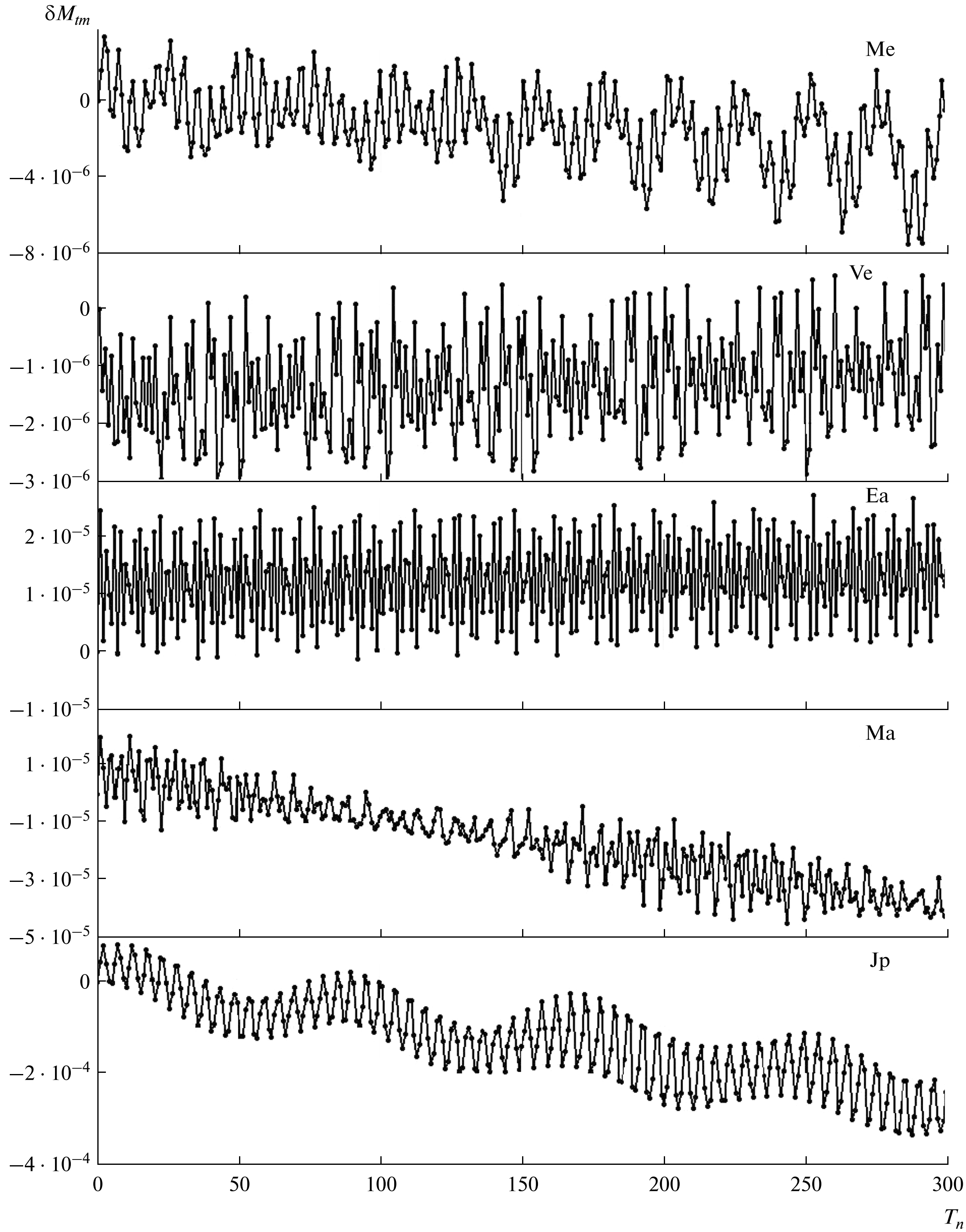

Fig. 4. Relative change of average modulus of angular momentum for planets from Mercury to Jupiter over 300 planetary rotations. The value of δM_{tm} was calculated for average momentum modulus M_{tm0} as of December 30, 1949.

rotation of bodies. These momenta are also referred to as spins. The above discussed programs for calculating orbital motion do not consider the spins of the bodies. Therefore, at this stage a study on changes of angular momentum in the dynamics of the Solar System can only be performed for orbital angular momenta.

It should be noted that the initial conditions in the Galactica system include, apart from orbital parame-

Table 3. Average relative differences of the DE405, DE403, and DE200 ephemeris and the Horizons system from the DE406 ephemeris

Source	Epoch Dec. 30, 1949		Epoch Nov. 30, 1999	
	δr_m	$\Delta\varphi_m$	δr_m	$\Delta\varphi_m$
DE405	1.0×10^{-11}	6.8×10^{-12}	1.0×10^{-11}	8.2×10^{-12}
DE403	2.1×10^{-7}	7.6×10^{-8}	3.0×10^{-7}	1.2×10^{-7}
DE200	8.6×10^{-7}	3.3×10^{-7}	3.2×10^{-6}	1.6×10^{-7}
Horizons	1.9×10^{-7}	1.5×10^{-7}	1.1×10^{-7}	5.2×10^{-8}

ters, the radii of the bodies and the projections of their spins. Therefore, if all of these parameters are specified for a problem of gravitational interaction of bodies, then solving this problem will give the dynamics of the orbital and rotational angular momenta of the bodies. This analysis may cover collisions of bodies, their mergers into one body, collisions of the merged bodies, and other processes accompanying collisions.

These processes are complex, and it is rather difficult to choose and develop algorithms to describe them. In this case, control over the measurements of the total (including the spins) angular momentum is the only reliable method to control the accuracy of the results.

It should be noted that this paper considers the change of angular momentum in the dynamics of the Solar System, i.e., in theories describing the motion of the Solar System. A change of the angular momentum of the Solar System depends not only on the orbital and rotational motion of bodies but also on other factors. The most important of them is orbital motion. In the future, with the increasing accuracy of the description of the first most important factors, the least important ones will also be taken into account.

Below we give an estimate for the momenta induced by the second most important factor, i.e., rotational motion of bodies. If J is the axial momentum of inertia and ω_{rt} is the angular velocity of rotation, then the spin of the body is

$$S = J \cdot \omega_{rt} \approx 0.4mR^2 \cdot 2\pi/P_{rt} = 0.8\pi m \cdot R^2/P_{rt},$$

where m is the mass of the body; R is its radius; and P_{rt} is its rotation period. If the average radius of the orbit is a and the angular velocity of the body's motion in orbit is ω_{or}, then its orbital angular momentum is $M = m \cdot \omega_{or} \cdot a^2 = 2\pi \cdot m \cdot a^2/P_{or}$, where P_{or} is the orbital period of the body. Then the ratio of the spin to the orbital momentum is written as

$$S/M = 0.4(R/a)^2 P_{or}/P_{rt}.$$

Table 4 presents these ratios for the planets (from Me to Pl) and the Moon (Mo). The Moon's orbital momentum was calculated for its orbit around the Earth, and the planets' momenta, for their orbits around the Sun. It is evident that the orbital momentum is many orders of magnitude greater than the spin. Nevertheless, the accuracy of Galactica appears to be able to take the latter into account. Thus, in the future researchers will be able to pose problems such as the one suggested by the reviewer and attempt to solve them using the Galactica system.

CONCLUSIONS

The accuracy of the existing methods for calculating the motion of space objects is inadequate for today's problems of space and celestial mechanics. For example, in order to improve the reliability of the calculated motion of Apophis after its encounter with the Earth in 2029, the accuracy of these methods should be increased by an order of magnitude [6, 7]. Researchers need more accurate methods, not only to calculate the motion of asteroids and spacecraft and to study the evolution of the Solar System over geological time intervals, but also for many other problems of celestial mechanics, e.g., to refine the masses of the planets. Our

Table 4. Parameters of the planets from Mercury to Pluto (Me to Pl) and the Moon (Mo) and their average orbital momenta (M) and spins (S). The "–" sign before the numbers indicates that the planet rotates clockwise

Body	$m \times 10^{-22}$, kg	R, thousand km	P_{rt}, days	a, million km	P_{or}, yrs	S, kg m^2/s	M, kg m^2/s	S/M
Me	33.019	2.4397	58.6462	57.909	0.2408	9.748×10^{29}	9.154×10^{38}	1.06×10^{-9}
Ve	486.86	6.0519	–243.01	108.21	0.6152	-2.134×10^{31}	1.845×10^{40}	-1.16×10^{-9}
Ea	597.37	6.3781	0.9973	149.60	1	7.088×10^{33}	2.662×10^{40}	2.66×10^{-7}
Ma	64.185	3.397	1.026	227.94	1.8807	2.1×10^{32}	3.530×10^{39}	5.95×10^{-8}
Jp	189900	71.492	0.4135	778.30	11.8565	6.827×10^{38}	1.932×10^{43}	3.53×10^{-5}
Sa	56860	60.268	0.4375	1429.4	29.4235	1.373×10^{38}	7.861×10^{42}	1.747×10^{-5}
Ur	8684.1	25.559	–0.65	2875.0	83.7474	-2.539×10^{36}	1.707×10^{42}	-1.49×10^{-6}
Ne	10246	24.764	0.768	4504.4	163.7230	2.38×10^{36}	2.528×10^{42}	9.41×10^{-7}
Pl	1.6509	1.151	–6.3867	5915.8	248.0208	-9.961×10^{28}	4.638×10^{38}	-2.15×10^{-10}
Mo	7.3477	1.738	27.3217	0.38440	0.0748	2.363×10^{29}	2.89×10^{34}	8.18×10^{-6}

studies on the change of angular momentum make it possible to assess the accuracy of the methods used for calculating motions and find the causes of their errors and the ways to improve these methods.

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