Exact Solution of Homographic Problem for Unlimited Number of Bodies

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There are exact solutions of the Newtonian gravitational interaction of N point particles with their certain configuration. For example, for N axisymmetric positions of bodies on a circle this task is fully solved [1]. In addition to a single-layer configuration in a number of works, for example [2], the exact solutions of several layers configurations are considered. Usually they are examined as a system of enclosed each other imagined polygons, rotating as a whole with angular velocity ω. At the vertices of the polygons are located point particles which interact with each other by Newton's law of gravitation. In generalizing work [2] the examples of solutions for the problems for the enclosed each other triangles, rhombuses, squares, pentagons and hexagons are given.

These problems have been solved [2] to multiple layers of polygons: for triangles - up to 4 layers, for squares - up to 3 layers, for pentagons and hexagons - up to 2 layers. The largest number of interacting bodies in the tops of these polygons was 12.

In frameworks of the Hamiltonian dynamics this problem is reduced to systems of algebraic equations that are solved by methods of computer algebra [2]. In each case the problem requires special consideration, and it is required to expend a lot of effort to solve it. These tasks are interacting point particles located at the vertices of nested polygons are studied as the problem homographic dynamics.

In this paper we consider some other approach to solving these problems. It is continuation of a method used in papers [1] and [3]. The movement of bodies is considered on the basis of the interaction forces between them. Instead of imaginary polygons, with which the particles are connected, we consider the circle on which there are particles. The ordering of parameters in work [3] was accepted for a basis of such structure. The axisymmetric multi-layer structure composed of N_2 circles, on each of which the N_3 bodies are located.

The forces of influence of all bodies on the first body on a ring j are deduced, and the differential equation of its motion is written in the trajectory coordinate system. As a result of the analysis it is established, that this equation will be identical for other bodies, if the angle of the first body on each ring $\varphi_{j,1}$ is 0 or half the angle between adjacent bodies on the ring. Then N_2 differential equations are transformed into N_2 algebraic equations. They describe rotating with angular velocity ω axisymmetric structure consisting of N_2 layers. Setting parameters of multilayer rotating structures are: N_2, N_3, $\varphi_{j,1}$, m0 is mass of the central body, ω, and the radii or the mass of peripheral bodies are defined by the N_2 equations.

The program RtCrcSt2.for is developed for the solution of algebraic equations. As a result of the solution of the problem the part of masses can be received negative. It is therefore necessary to choose the parameters so that the masses were positive. Program RtCrcSt2.for allows choosing necessary parameters determining structure. In addition, this program prepares the file of source data and the initial conditional for the program Galactica, which is in a free access [4]. Galactica system allows computing the dynamics of a rotating multi-layer structure and examining its evolution. This should be done in the further use of such structures.

Completed studies in the frame of force interaction have shown that all multi-layers rotating structures are unstable. However, the lifetime of the structure can vary widely. When considering the structure parameters in dimensionless units, its lifetime is more, than its relative velocity of rotation is less. At a certain velocity, the accuracy of the calculation of the structure parameters affects on the lifetime. In addition, if such structure is among other influencing bodies, then the peripheral bodies begin to fluctuate. It violates their tendency to converge, and the lifetime of the structure can be increased [5].

During the solution of this problem the multi-layers rotating structure have been calculated with a number of rings: 1, 2, 3, 4, 5, 15, 30, 100, 103 and 1000. On each ring the number of bodies was set 2, 5, 8, 10, 29, 30 and 999. The configurations created with different variants of the first body angle $\varphi_{j,1}$ on each ring, as well as its various alternating on adjacent rings. The structures were calculated that are formed at different initial masses. As initial mass the mass of the Earth and of the Sun were set. The structures with the central body and without it were examined.

Most of the homographic problems presented nested regular polygons [2]. However, there are irregular polygons, such as nested rhombus. It is shown that such a multi-layer rhombic structure is created as a series of rings, each of which contains two bodies and the angles of the first bodies on the rings alternate.

References

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