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The Evolution of the Moon's Orbit Over 100 Million Years and Prospects for the Research in the Moon

Joseph J. Smulsky

Abstract

As a result of solving the problem of interaction of Solar-system bodies, data on the evolution of the Moon's orbit were obtained. These data were used as the basis for the development of a mathematical model for the Moon representing its motion over an interval of 100 million years. A program of exploration of the Moon with the aim of creating a permanent base on it is outlined. Such a base is intended for exploring the Earth, the Sun, and outer space.

Keywords: Moon, orbit, evolution, exploration, life, investigation, Earth, Sun, space

1. Introduction

The Moon is a satellite of the Earth; therefore, it is the body closest to our planet. After the Sun, the Moon exerts the second greatest impact on the Earth. For these reasons, the Moon will occupy the first place in the future exploration of space.

Since the Moon always faces the Earth from one side, its main motion is orbital. Therefore, all the features of this motion are of interest, including its evolution over long time intervals. The first part of this chapter is devoted to this problem.

In the second part, prospects for further space research are considered. More than half a century of experience in this research has shown that the effectiveness of studies strongly depends on the resource base invoked for performing the studies. Of the celestial bodies, the Moon is the body most suitable for creating a base on it. The chapter discusses a wide range of issues related to the feasibility of creating such a base, its structure, functioning, and prospects for research on it.

2. Evolution of the Moon's orbital motion

2.1 Coordinate system and orbital parameters

When solving the problem of the evolution of the rotational motion of the Earth over millions of years [1], it is necessary to have data on the coordinates of bodies acting on the Earth at any time from this time interval. The Moon exerts two-thirds

of the total influence exerted on the Earth's rotational motion. The evolution of the Moon's orbital motion is therefore an important component of the posed problem.

The evolution of the orbits of Solar-system bodies can be determined by solving the problem of interaction for those bodies. For solving this problem, a Galactica system was developed [2–4]. The accuracy ensured by this software is several orders of magnitude higher than the accuracy ensured by other similar systems [5, 6]; this made it possible to solve the problem of the evolution of the Solar-system over 100 million years [7]. This problem is solved in the barycentric coordinate system *xyz* (**Figure 1**) attached to the fixed equatorial plane A_0A_0'. The origin O is located at the center of mass of the Solar-system. The results of solving this problem were saved in files following each 10 thousand years. Then, for those epochs, the interaction problem was solved, with the help of the Galactica program, per one orbital revolution of the body, and 11 parameters of its orbit were determined from its coordinates. For the Moon, the parameters of its orbit relative to the Earth are to be determined.

The Moon's orbital period is very short compared to that of planets. Therefore, the oscillation periods of the parameters of the Moon's orbit repeated many times over an interval of 10 thousand years. Therefore, with this interval, the evolution of the Moon's orbit was studied during its 736 continuous revolutions around the Earth, which took place during 56.7 years.

The position of the Moon's orbital plane, $\gamma_{Mo}A_1B$, is specified by its angle of inclination i_{Mo} to the plane of the stationary equator A_0A_0' and by the angle of the ascending angle $\varphi_{\Omega Mo}$, both defined in the caption to **Figure 1**. The position of the perigee of the Moon's orbit is specified by the angle φ_p. When analyzing the Moon's orbit, the origin O is located at the center of the Earth.
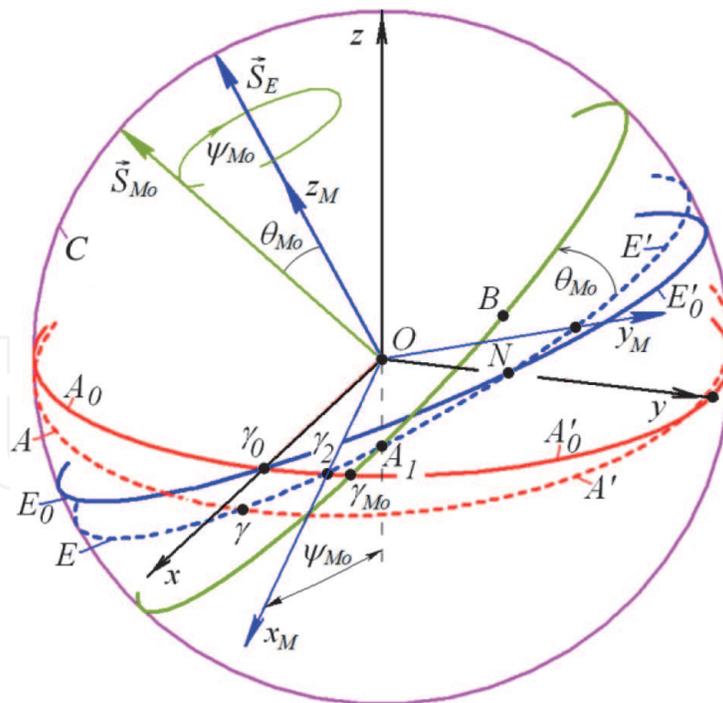
Figure 1.
The coordinate system and the main characteristics of the Moon's orbit: C is the celestial sphere; A_oA_o' and E_oE_o' are the fixed planes of the equator and the Earth's orbit (ecliptic) for the epoch of 2000.0, $JD_S = 2451545$; AA' and EE' are the moving planes of the Earth's equator and orbit as of the current date; $\gamma_{Mo}A_1B$ is the Moon's orbital plane; $\vec{S}_E$ and $\vec{S}_{Mo}$ are the axes of the Earth's and Moon's moving orbits, respectively EE' and $\gamma_{Mo}A_1B$, which are perpendiculars to the orbital planes. The angles of the Earth's orbital plane relative to its equatorial plane A_oA_o': $\varphi_{\Omega E} = \gamma_o\gamma_2$, $i_E = A_1\gamma_2\gamma_{Mo}$; and same angles of the Moon's orbital plane relative to the Earth's equatorial plane: $\varphi_{\Omega Mo} = \gamma_o\gamma_{Mo}$, $i_{Mo} = A_1\gamma_{Mo}A_o'$. B is the projection of the perigee of the Moon's orbit on the celestial sphere, and $\varphi_p = \gamma_{Mo}B$ is its angular position; ψ_{Mo} u θ_{Mo} are the precession and inclination angles of the Moon's orbit relative to the moving plane of the Earth's orbit EE'.

2.2 Dynamics of the Moon's orbit in the initial epoch

In the initial epoch T = 0, on the date of December 30, 1949 with the Julian-day number JD_0 = 2433280.5, consider the variation of the Moon's orbital parameters on a doubled interval of $\pm$ 736 revolutions, or $\pm$ 56.7 years. In order to distinguish between fluctuations, the results in **Figure 2** are shown for an interval of $\pm$10 years. The perigee radius R_p oscillates with a period T_{Rp} =0.5637 years around its average value R_{pm} = 3.622069 $\cdot$ 10^5 km. The eccentricity of the orbit e oscillates with the same period around the mean value e_m = 0.0563331. In addition, there is a longer period of 3.719 years, yet exhibiting smaller oscillation amplitude.

The period of revolution of the Moon around the Earth P with respect to fixed stars, i.e. the sidereal period, oscillates around the average value P_m = 7.47928 $\cdot$ 10^{-2} years. There are two oscillation periods lasting 0.664039 and 3.719 years.

Over the entire interval, the perigee angle φ_p almost linearly increases into the future i.e. the perigee of the Moon's orbit rotates counterclockwise. The sidereal period of this rotation is $T_{\varphi p}$ = 8.8528 years. In addition, the perihelion angle
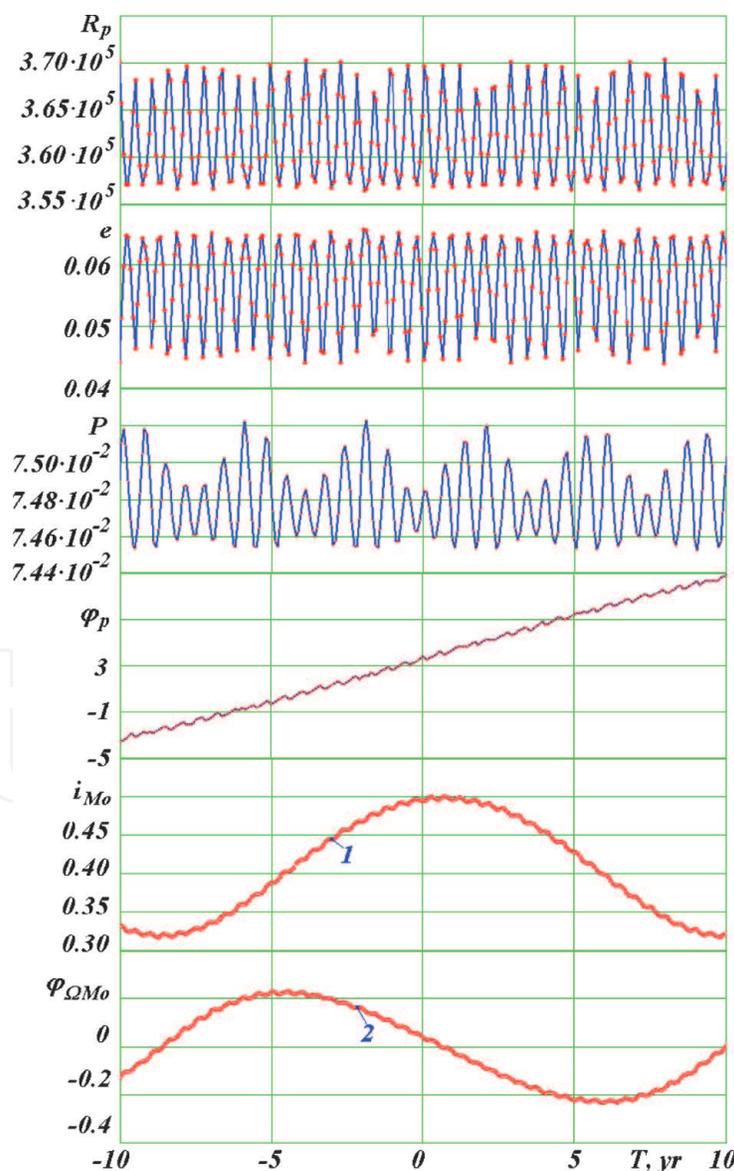
Figure 2.
*Dynamics of the Moon's orbital elements in the geocentric equatorial coordinate system: perigee radius R$_p$ – in km, period P and time T – in sidereal years with a duration of 365.25636042 days, angles φ$_p$, i$_{Mo}$ and φ$_{Mo}$ – in radians; the white centerlines 1 and 2 are the approximating dependences (13) and (16), respectively. For other designations, see **Figure 1**.*

oscillates with a short period $T_{\varphi p1}$ = 0.5637 years and with a long period $T_{\varphi p2}$ =18.6006 years.

The inclination angle i_{Mo} oscillates around its mean value i_{Mom} = 0.41526. The oscillations occur with two periods: a short one, of 0.4745 year, and a long one, of 18.6006 years. The angle of the ascending node $\varphi_{\Omega Mo}$ oscillates around the mean value of $\varphi_{\Omega Mom}$ = 6.5472 · 10^{-4} with the same periods.

2.3 Precession of the Moon's orbital axis

When studying the orbits of the planets, we introduced the orbital axis $\vec{S}$ in the form of a unit-length vector normal to the orbital plane [8]. Using the inclination angle i_{Mo} and the ascending-node angle $\varphi_{\Omega Mo}$, we can write the projections of the orbital axis onto the axes of the xyz-coordinate system:

$$S_{Moz} = \cos i_{Mo}; \quad S_{Moy} = \sqrt{1 - S_{Moz}^2} \cos \varphi_{\Omega Mo}; \quad S_{Mox} = -S_{Moy} \operatorname{tg} \varphi_{\Omega Mo} \tag{1}$$

The orbital axes of all planets precess about the angular momentum of all Solar-system bodies. As a result of the study, it was found that the axis $\vec{S}_{Mo}$ of the Moon's orbit precesses about the moving axis $\vec{S}_E$ of the Earth's orbit (**Figure 1**). The same will also be shown below. We introduce a coordinate system $x_M y_M z_M$. Along the z_M-axis of this system, the axis $\vec{S}_E$ is directed, and the axis x_M passes through the ascending node γ_2 of the Earth's orbit. Then, using the angles i_E and $\varphi_{\Omega E}$ specifying the position of the Earth's orbital plane and the projections of the Moon's orbital axis according to formulas (29) in Melnikov and Smulsky [7], one can find the projections S_{MoxM}, S_{MoyM}, and S_{MozM} of the axis $\vec{S}_{Mo}$ onto the axes of the $x_M y_M z_M$-coordinate system. **Figure 3a** shows the motion of the endpoint of the orbital axis $\vec{S}_{Mo}$ as projected onto the $y_M x_M$-plane over the examined time interval of 113.4 years. It is evident from the graph that the endpoint of the vector $\vec{S}_{Mo}$ moves in a circle with slight fluctuations. The rotation period is T_S = −18.6006 years, and the oscillation period is $T_{\psi 1}$ = 0.4745 years. During the time interval under consideration, the axis $\vec{S}_{Mo}$ makes six revolutions in the clockwise direction.

From the projection onto the $z_M x_M$-plane (**Figure 3b**), it can be seen that the endpoint of the vector $\vec{S}_{Mo}$ executes small-amplitude oscillations along the z_M-axis with a swing of Δz_M = 4.43 · 10^{-4}. Those oscillations are symmetrical about the x_M-axis.

Thus, the Moon's orbital axis precesses in a clockwise direction relative to the Earth's orbital axis. The precession period T_S is 18.6006 years. The precession proceeds with oscillations, which are called nutational. The period of the latter oscillations is $T_{\psi 1}$ = 0.4745 years.

Such studies were carried out for each epoch following 10 thousand years over time intervals of 0 ÷ −2 million years and −98 ÷ −100 million years. As a result, it was found that in all these cases the Moon's orbital axis $\vec{S}_{Mo}$ precesses relative to the moving axis $\vec{S}_E$ of the Earth's orbit (**Figure 1**).

In the $x_M y_M z_M$-coordinate system, the Moon's orbital axis $\vec{S}_{Mo}$ (**Figure 1**) is specified by the inclination and precession angles (respectively, θ_{Mo} and ψ_{Mo}):

$$\theta_{Mo} = \arccos S_{MozM}; \; ... \psi_{Mo} = \operatorname{arctg} S_{MoyM}/S_{MoxM} + 0.5\pi \tag{2}$$
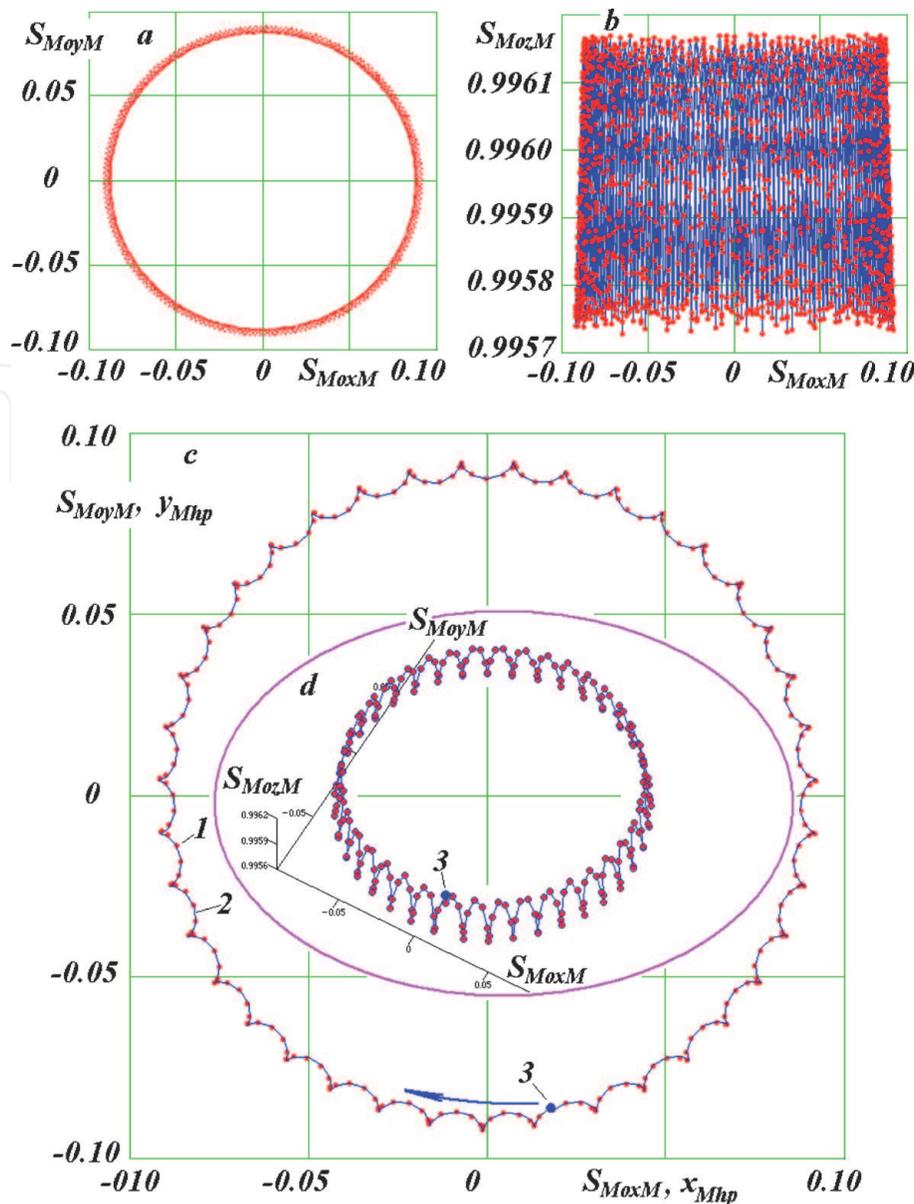
Figure 3.

Projections of the Moon's precession axis $\vec{S}_{Mo}$ onto the axes of the $x_M y_M z_M$ coordinate system over a period of 113.4 years (a, b) and from T = 0 to T = 18.6 years (c, d): d – in three-dimensional form; points: (1) projection of the axis $\vec{S}_{Mo}$ onto the $y_M x_M$-plane; (2) hypocycloid equation $y_{Mhp}(x_{Mhp})$ (4)–(5); and (3) position of the axis $\vec{S}_{Mo}$ at T = 0.

Since the precession angle ψ_{Mo} varies over ranges wider than 2π, for calculating continuous values of this angle, its values at adjacent time intervals were calculated, and then the values obtained were summed using certain rules. As a result of studying the variation of the angle ψ_{Mo}, it was found that this angle decreases into the future, i.e., the axis $\vec{S}_{Mo}$ rotates in the clockwise direction with the rotation period of $T_S = -18.6006$ years. In addition, the angle ψ_{Mo} oscillates with a period of $T_{\psi 1} = 0.4745$ years and an amplitude of $\Delta\psi_{MoA} = 0.023662$. The inclination angle also oscillates with the latter period and with an amplitude $\theta_{MoA} = 0.002464 = 8.4692'$ about the mean value $\theta_{Mom} = 0.09006 = 5.1544°$.

Figure 3c shows, on a larger scale, the projection of the precessing orbital axis $\vec{S}_{Mo}$ onto the $y_M x_M$-plane for one precession period $T_S = -18.6006$ years. Starting from the moment $T = 0$, marked by point 3, the end of the axis moves clockwise. It is evident from the graph that the endpoint of the vector $\vec{S}_{Mo}$ moves exactly around a circle, and the nutational oscillations here are regular. In **Figure 3d** dots and a line

show the precession of the orbital axis in three dimensions. On the graph, the scale along the vertical z_M axis is increased 40 times.

As a result of an analysis, it was found that the endpoint of the vector $\vec{S}_{Mo}$ moves exactly along a hypocycloid. The hypocycloid is formed by some point of a circle of radius r rolling without slippage on the inner side of another circle of radius R.

In the $y_M x_M$-plane (**Figure 3c**), the radius of the great circle $R = \sin\theta_{Mom}$ is the mean value of the projection of the orbital axis $\vec{S}_{Mo}$ onto this plane, and the radius of the small circle $r = \sin\theta_{MoA}$ is the oscillation amplitude of this axis. The center of the small circle moves in the clockwise direction with the angular velocity $2\pi/T_S$. In this translational motion, the oscillations of the vector $\vec{S}_{Mo}$ occur with the period $T_{\psi 1}$, or with the angular velocity $2\pi/T_{\psi 1}$. Then, the absolute angular velocity of rotation of the small circle, $2\pi/T_n$, will be equal to the sum of these velocities: $2\pi/T_n = 2\pi/T_{\psi 1} + 2\pi/T_S$. That is why the period of the nutational rotation will be

$$T_n = \frac{T_S \cdot T_{\psi 1}}{T_S + T_{\psi 1}} = 0.48692 \text{ years} \tag{3}$$

Then, in the $y_M x_M$-plane the equation of the hypocycloid can be written as follows:

$$x_{Mhp} = R \cos\left(\varphi_{10} + 2\pi \frac{T}{T_S}\right) + r \cos\left(\varphi_{20} + 2\pi \frac{T}{T_n}\right) \tag{4}$$

$$y_{Mhp} = R \sin\left(\varphi_{10} + 2\pi \frac{T}{T_S}\right) + r \sin\left(\varphi_{20} + 2\pi \frac{T}{T_n}\right) \tag{5}$$

where $\varphi_{10} = 4.92766$ and $\varphi_{20} = 2.19315$ are the initial phases that specify the position of the vector $\vec{S}_{Mo}$ on the circles at the initial time $T = 0$.

The line in **Figure 3c** shows the trajectory of the motion along the hypocycloid, given by Eqs. (4) and (5), and the points are the projection of the motion of the Moon's orbital axis. Both are perfectly coincident. Thus, the Moon's orbital axis $\vec{S}_{Mo}$ executes an averaged clockwise motion around the Earth's orbital axis $\vec{S}_E$ with a period $T_S = -18.6006$ years. Here, the average angle between the axes $\vec{S}_{Mo}$ and $\vec{S}_E$ is $\theta_{Mom} = 5.1544°$.

The orbital axis $\vec{S}_{Mo}$ executes a second counterclockwise rotational motion about the averaged motion with a period T_n and an angular deviation from the median axis $\theta_{MoA} = 8.4692'$. During the complete revolution of the averaged axis of the Moon's orbit, $-T_S/T_{\psi 1} = 39.2$ nutational revolutions of the instantaneous axis $\vec{S}_{Mo}$ occur.

The dynamics of the inclination and precession angles, θ_{Mo} and ψ_{Mo}, of the Moon's orbit over an interval of 20 years is shown in **Figure 4**. The oscillations of the angle θ_{Mo} are more regular than those of the angle i_{Mo} of inclination of the Moon's orbit to the equatorial plane. They are harmonic with one and the same oscillation period. The precession angle ψ_{Mo} executes similar oscillations. At the same time, it monotonically decreases, this decrease being indicative of a clockwise precession of the orbital axis $\vec{S}_{Mo}$. The light line shows the approximating time dependence of the precession angle

$$\psi_{Mo}(T) = \psi_{Mo0} + 2\pi \cdot T/T_S + \Delta\psi_0 \tag{6}$$

where $\psi_{Mo0} = 0.202798$, and $T_S = -0.186006$ is the period of precession of the Moon's orbital axis $\vec{S}_{Mo}$.
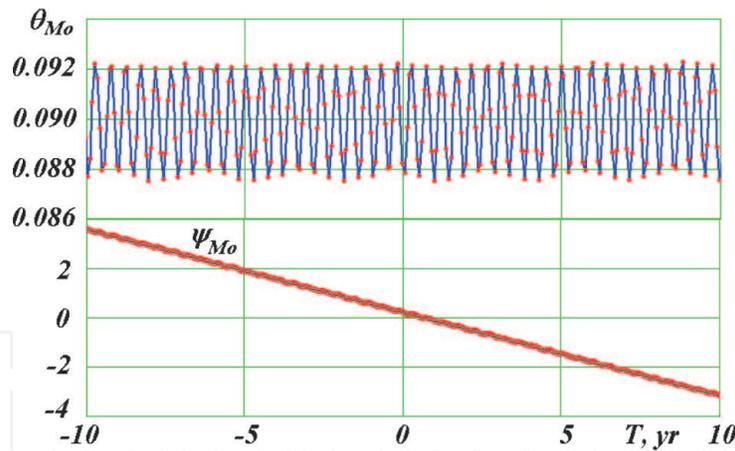
Figure 4.
Dynamics of the inclination and precession angles, θ_{Mo} and ψ_{Mo} of the Moon's orbit relative to the Earth's moving orbit. In the graph of ψ_{Mo}, the line shows the approximating dependence (7).

Changes in the Moon's orbit occur in the form of two groups of motions. In the first group, changes occur in the orbital plane with variation of the following parameters: perigee radius $R_p(T)$, perigee angle $\varphi_p(T)$, orbital eccentricity $e(T)$, and the orbital period $P(T)$. Here, the character T denotes the time dependence of the elements. In the second group, changes occur of the Moon's orbital plane $\gamma_{Mo}A_1B$ (**Figure 1**), specified by the angles $\varphi_{\Omega E}$ and i_{Mo} relative to the equatorial plane A_0A_0' or by the angles ψ_{Mo} and θ_{Mo} relative to the moving plane of the Earth's orbit EE'. Since the latter angles change more regularly, using them is more preferable in describing the motion of the Moon's orbital plane.

2.4 Approximation of the orbital-plane elements

As it was noted above, the behavior exhibited by the Moon's orbital elements was studied for the period of 736 its continuous revolutions in different epochs over the interval from 0 to 100 million years. In addition, the elements of the Moon's orbit were investigated following the adoption of different initial conditions in the integration of the equations of motion using the Galactica program [7]. As a result of these studies, regularities of the dynamics of the elements were established, and the approximating dependences for them were chosen. The final form of the approximations was refined on a doubled interval from -736 to $+736$ revolutions, in which the meantime falls onto the epoch of December 30.0, 1949 with the Julian-day number JD_0 = 2433280.5. The perigee radius is defined by the expression

$$R_p(T) = R_{pm} + R_{pA} \cdot \sin\left(\varphi_{Rp0} + 2\pi \cdot T/T_{Rp}\right) \tag{7}$$

where R_{pm} = 3.622069 · 10^5 km is the average value of the perigee radius, R_{pA} = 6.2754 · 10^5 km is the amplitude of oscillations, φ_{Rp0} = 0.942478 is the initial phase of oscillations, and T_{Rp} = 0.005637 is the period of oscillations of the perigee radius. The time T and the periods of oscillations are expressed here in sidereal centuries of 36525.636042 days per century, and they are counted from the JD_0 epoch, December 30, 1949.

As it is evident from the graph $R_p(T)$ in **Figure 2**, there are oscillation beats of the perigee radius, which can be described by invoking a second harmonic with a large period. However, due to the irregularity of these beats over large time intervals, the second harmonic did not significantly improve the approximation of the perigee radius.

The eccentricity is approximated with two harmonics:

$$e(T) = e_0 + e_{A1} \cdot \sin\left(\varphi_{e01} + 2\pi \cdot T/T_{e1}\right) + e_{A2} \cdot \sin\left(\varphi_{e02} + 2\pi \cdot T/T_{e2}\right) \qquad (8)$$

whose characteristics are given in **Table 1**.

The perigee of the Moon's orbit rotates counterclockwise and, in addition, it executes oscillatory movements, which were also approximated with two harmonics:

$$\varphi_p(T) = \varphi_{p0} + 2\pi \cdot T/T_{\varphi p} + \Delta\varphi_{p01} + \varphi_{pA1} \cdot \sin\left(\varphi_{p01} + 2\pi \cdot T/T_{\varphi p1}\right) + \Delta\varphi_{p02} + \varphi_{pA2}$$
$$\cdot \sin\left(\varphi_{p02} + 2\pi \cdot T/T_{\varphi p2}\right)$$

$$(9)$$

where $T_{\varphi p}$ is the period of revolution of the Moon's perigee, and $T_{\varphi p1}$ and $T_{\varphi p2}$ are the first and second periods of oscillations of the perigee angle. The coefficients entering Eq. (9) are given in **Table 2**.

The Moon's orbital period P oscillates around its mean value P_m. The analysis of these oscillations was carried out considering the relative difference $\delta P = (P - P_m)/P_m$. Since the period P and the semi-major axis a of the Moon's orbit vary consistently, the analysis of those oscillations in relative differences allows their consistent approximation. The period P is also approximated with two harmonics:

$$P(T) = P_m \left[\left(1 + \Delta P_0 + \Delta P_{A1} \cdot \sin\left(\varphi_{pr01} + 2\pi \cdot T/T_{p1}\right) + \Delta P_{A2} \cdot \sin\left(\varphi_{pr02} + 2\pi \cdot T/T_{p2}\right)\right)\right]$$

$$(10)$$

where T_{p1} and T_{p2} are the first and second oscillation periods of the orbital period, and the values of the coefficients are given in **Table 3**.

e_0	e_{A1}	φ_{e01}	T_{e1}
0.0563331	0.0113634	−2.19911	0.005637
	e_{A2}	φ_{e02}	T_{e2}
	6.91384 E−4	−1.5708	0.03719

Table 1.
Coefficients in Eq. (8).

φ_{p0}	$T_{\varphi p}$	$\Delta\varphi_{p01}$	φ_{pA1}	φ_{p01}	$T_{\varphi p1}$
3.67159	0.088528	1.34024E−4	0.200529	2.19911	0.005637
		$\Delta\varphi_{p02}$	φ_{pA2}	φ_{p02}	$T_{\varphi p2}$
		−4.91312E−3	0.196967	−0.188496	0.186006

Table 2.
Coefficients in Eq. (9).

P_m	ΔP_0	ΔP_{A1}	φ_{pr01}	T_{p1}
7.479277E−4	1.01403E−4	0.00385003	0.628319	0.00664039
		ΔP_{A2}	φ_{pr02}	T_{p2}
		0.00141509	−1.41372	0.03719

Table 3.
Coefficients in Eq. (10).

Evidently, some parameters have identical oscillation periods. Perigee radius $R_p(T)$, eccentricity $e(T)$, and perigee angle $\varphi_p(T)$ have identical first periods of 0.005637 century, whereas the eccentricity and the orbital period $P(T)$ have identical second oscillation period of 0.03719 century.

2.5 Approximation of the orbital angles

As a result of studies, it was found that the precession angle ψ_{Mo} oscillates with two periods, a shorter (0.4745 years) and a longer (2.995 years) one. Since the amplitude of the large-period oscillations is small, we neglect those oscillations. As a result, the precession angle can be approximated with the following expression:

$$\psi_{Mo}(T) = \psi_{Mo0} + 2\pi \cdot T/T_S + \Delta\psi_{Mo0} + \Delta\psi_{MoA} \cdot \sin\left(\varphi_\psi + 2\pi \cdot T/T_{\psi 1}\right) \quad (11)$$

where ψ_{Mo0} = 0.202798, $T_S = -0.186006$ is the precession period of the Moon's orbital axis $\vec{S}_{Mo}$, $\Delta\psi_{Mo0}$ = $2.30247 10^{-4}$, $\Delta\psi_{MoA}$ = 0.023662, φ_ψ = 2.82743, and $T_{\psi 1}$ = 0.004745 is the period of oscillations of the precession angle ψ_{Mo}.

The inclination angle θ_{Mo} also oscillates with two periods. The longer period, equal to 2.995 years, has an amplitude of $5.978 \cdot 10^{-5}$ radians, which value is almost two orders of magnitude smaller than the amplitude of the first period. Therefore, the second harmonic, i.e. the one with the period of 2.995 years, can also be neglected, and the approximation for the nutation angle, therefore, has the form:

$$\theta_{Mo}(T) = \theta_{Mo0} + \theta_{MoA} \cdot \sin\left(\varphi_\theta + 2\pi \cdot T/T_{\psi 1}\right) \quad (12)$$

where θ_{Mo0} = 0.09006, θ_{MoA} = 0.002464, and φ_θ = -2.19911

The angles ψ_{Mo} and θ_{Mo} are tied to the moving plane of the Earth's orbit EE' (see **Figure 1**), so they are inconvenient to use. We, therefore, pass to the angles $\varphi_{\Omega Mo}$ and i_{Mo}, which specify the position of the Moon's orbital plane relative to the fixed plane of the equator A_0A_0' (**Figure 1**), with which the main coordinate system xyz is associated. In the spherical triangle $\gamma_2\gamma_{Mo}A_1$, the side $\gamma_2A_1 = \psi_{Mo}$ and the two angles $\gamma_2 = i_E$ and $A_1 = \theta_{Mo}$ are known. The cosine theorem can be used to determine the obtuse angle $\gamma_2\gamma_{Mo}A_1$, from which the acute angle i_{Mo} can be subsequently found: $i_{Mo} = \pi - \gamma_2\gamma_{Mo}A_1$. As a result, for the angle of inclination of the Moon's orbital plane to the plane of the stationary equator, we obtain the following expression:

$$i_{Mo} = \pi - \arcos\left(-\cos i_E \cdot \cos\theta_{Mo} + \sin i_E \cdot \sin\theta_{Mo} \cdot \cos\psi_{Mo}\right) \quad (13)$$

As it is seen from **Figure 1**, the angle specifying the position of the ascending node of the Moon's orbit is equal to the sum of two arcs,

$$\varphi_{\Omega Mo} = \gamma_0\gamma_{Mo} = \varphi_{\Omega E} + \gamma_2\gamma_{Mo} \quad (14)$$

By the sine theorem, in the triangle $\gamma_2\gamma_{Mo}A_1$ we have

$$\sin\gamma_2\gamma_{Mo}/\sin\theta_{Mo} = \sin\psi_{Mo}/\sin\left(\pi - i_{Mo}\right) \quad (15)$$

and, therefore, the arc $\gamma_2\gamma_{Mo}$ can be found. Then, according to Eq. (14), the position of the ascending node can be found as

$$\varphi_{\Omega Mo} = \varphi_{\Omega E} + \arcsin\left[\sin\psi_{Mo} \cdot \sin\theta_{Mo}/\sin\left(\pi - i_{Mo}\right)\right] \quad (16)$$

In order to check the validity of the obtained approximations of the Moon's orbital elements (13) and (16), we superimposed onto **Figure 2** the calculated
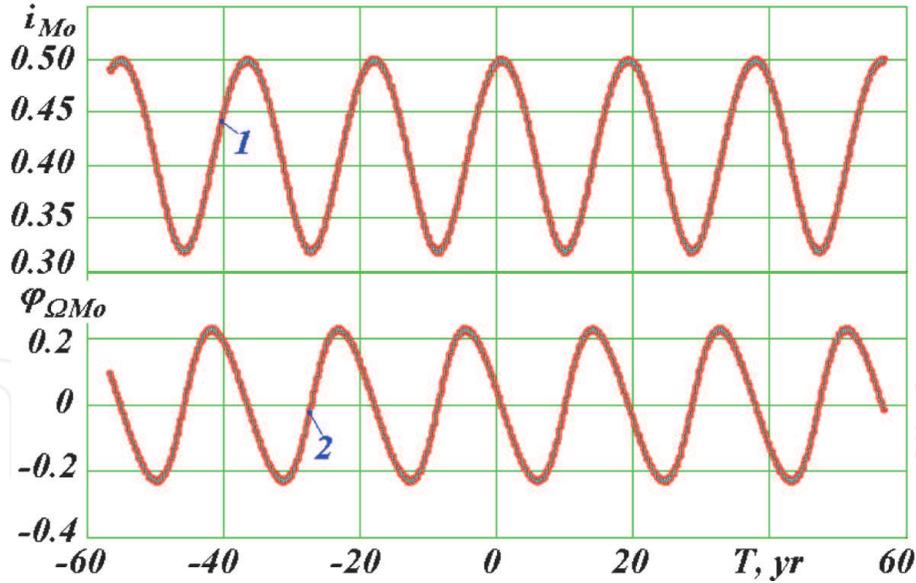
Figure 5.
Comparison of the dynamics of the angles i_{Mo} *and* $\varphi_{\Omega Mo}$, *specifying the position of the Moon's orbital plane relative to the equatorial plane, as obtained in two ways: thick lines – numerical integration; light thin lines 1 and 2 – approximating dependences (13) and (16), respectively.*

elements that were obtained using the Galactica program for the integration of the equations of motion. **Figure 5** shows, over the entire interval of $\pm$ 56.7 years, the dynamics of the angles i_{Mo} and $\varphi_{\Omega Mo}$ obtained by two methods: using numerical integration (thick lines) and using approximations (13) and (16) (light line). As it is seen from the graphs, the approximations yield data perfectly coincident with the short- and long-period oscillations of the angles i_{Mo} and $\varphi_{\Omega Mo}$. Thus, this check has fully confirmed the validity of the adopted approximations.

2.6 Evolution of orbital elements over an interval of 100 million years

So, the dynamics of Moon's orbital elements R_p, e, φ_p, P, i_{Mo}, and $\varphi_{\Omega Mo}$ relative to the fixed plane of the equator in the geocentric coordinate system *xyz* is described by Eqs. (7)–(10), (13), and (16). This description was obtained over a time interval of 113.4 years. As already mentioned, for establishing the validity of this description over large time intervals, studies were carried out over intervals of $0 \div -2$ million years and $-98 \div -100$ million years. Following each 10 thousand years, the dynamics of the Moon's orbital elements were investigated during 736 continuous orbital revolutions of the Moon. The dynamics in different epochs did not differ qualitatively from that shown in **Figure 2**. With the purpose of comparison of those dynamics, the average values of individual elements during 736 orbital revolutions were calculated. Then, the evolution of these average values, as well as the periods of rotation, periods of oscillations, and oscillation amplitudes overtime periods of 2 million years with an interval between points of 10 thousand years, was investigated.

As an example, **Figure 6** shows the evolution of the average orbital period P_m, eccentricity e_m, inclination angle θ_{Mom}, and the amplitude θ_{MoA} of nutational oscillations. The graphs show the relative changes of these quantities. These changes were determined in the same way, for example, for the average period of the Moon's orbital revolution this value was calculated as follows:

$$\delta P_m = (P_m - P_{m0})/P_{m0} \tag{17}$$

where P_{m0} is the value of the average orbital period over 736 orbital revolutions of the Moon in the modern epoch. In calculating the relative changes of the
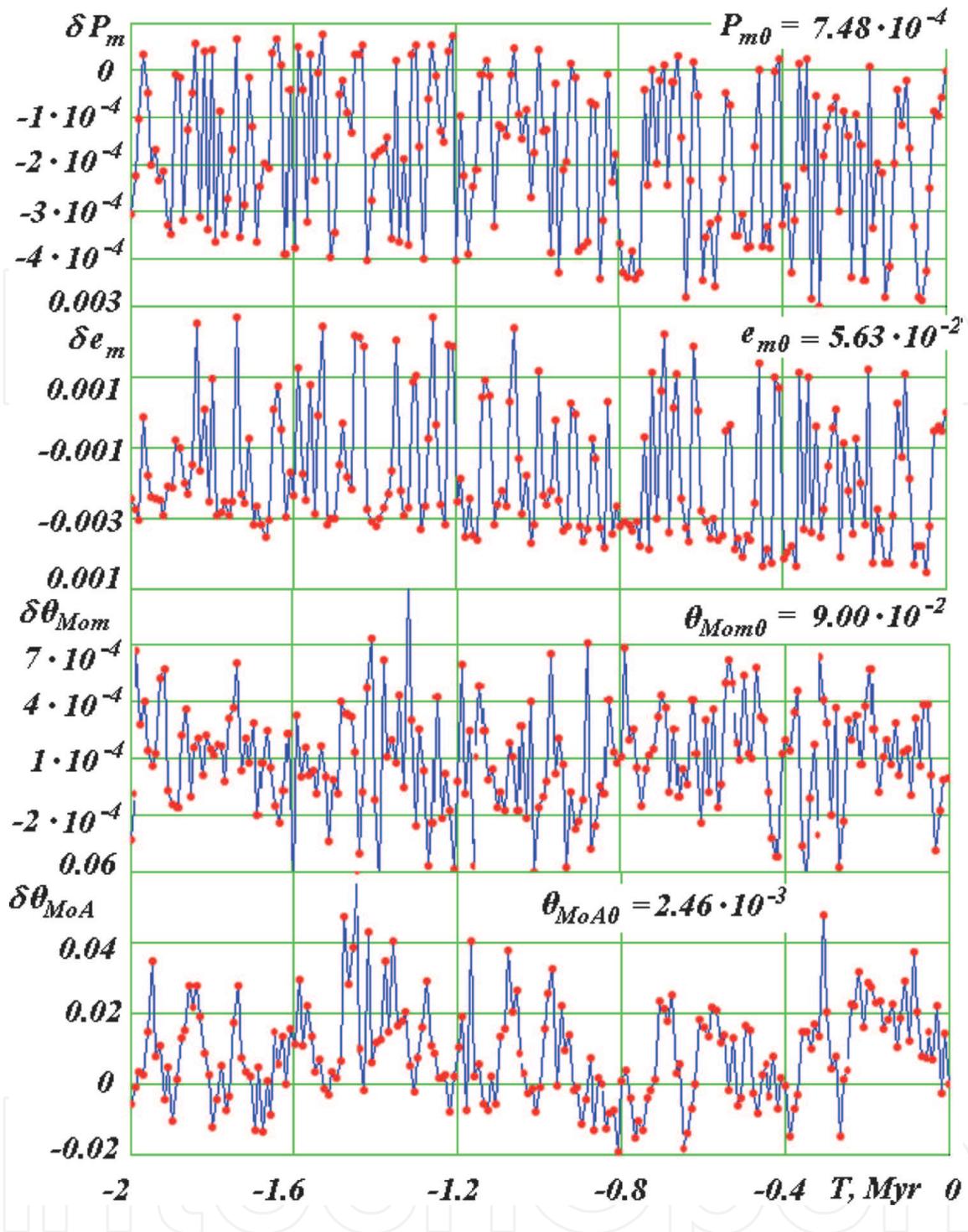
Figure 6.
Evolution, over the period of 2 million years, of relative averages for 736 revolutions of the deviations of Moon's orbital parameters: period δP_m, eccentricity δe_m, inclination angle $\delta\theta_{Mom}$, and the amplitude of nutational oscillations $\delta\theta_{MoA}$; T – in million years.

amplitudes ($\delta\theta_{MoA}$), instead of the mean values entering Eq. (17), the amplitude θ_{MoA} yielded by approximation (12) was used.

As it is seen from **Figure 6**, the oscillation amplitude of the relative mean δP_m, δe_m and $\delta\theta_{Mom}$ are $2\cdot10^{-4}$, 0.003, and $4.5\cdot10^{-4}$, respectively. At the same time, the similar relative oscillation amplitudes during 736 Moon's orbital revolutions are $3.85\cdot10^{-3}$, 0.2, and $2.7\cdot10^{-2}$, respectively. Thus, the analyzed fluctuations of Moon's parameters P, e, and θ_{Mo} exceed their changes over the interval of $0 \div -2$ million years by factors of 19, 67, and 60, respectively. This conclusion is also confirmed by the graph $\delta\theta_{MoA}(T)$ in **Figure 6**: over the interval of $0 \div -2$ million years, the amplitude of nutational oscillations θ_{MoA} fluctuates within 2%.

The rest approximation parameters exhibit similar behavior. Similar results were obtained for the interval of $-98 \div -100$ million years. This allows us to conclude that, over the interval of $0 \div -100$ million years, if there occur oscillations with longer periods than those used in our approximations, then the amplitude of such oscillations does not exceed a few percent of the considered oscillation amplitudes.

2.7 Mathematical model for the Moon's motion

Thus, Eqs. (7)–(10), (13), (16) describe the evolution of Moon's orbital elements R_p, e, φ_p, P, i_{Mo}, and $\varphi_{\Omega Mo}$ which can be used over the interval $0 \div -100$ million years. We have developed a mathematical model of body motion in an elliptical orbit [9], which is based on the listed orbital elements. That is why this model, with Eqs. (7)–(10), (13), and (16), allows one to calculate the Moon's coordinates in the equatorial coordinate system at any time in the interval of $0 \div -100$ million years.

Figure 7 compares the Moon's orbits calculated using this model with a time step of $1 \cdot 10^{-4}$ years and numerical integration performed with the help of the Galactica program. The same orbital comparisons were made for the planets [9]. The orbits of the planets calculated by the mathematical model are no visual difference from the orbits obtained by numerical integration. As it is seen from **Figure 7**, such differences are observed for the Moon's orbit. This is due to the shorter Moon's orbital period compared to that of the planets. Nevertheless, this mathematical model of the Moon made it possible to solve the problem of the evolution of the Earth's rotational axis with acceptable accuracy. Comparison of the results of this problem for 200 thousand years, solved with this model of the Moon's orbit and without it, proved differences to be insignificant [1].

2.8 Comparison of calculations with observation data

The orbital periods of the Moon, the precession of its orbital axis, and the rotation of the perihelion oscillate about the average values of these quantities. Over
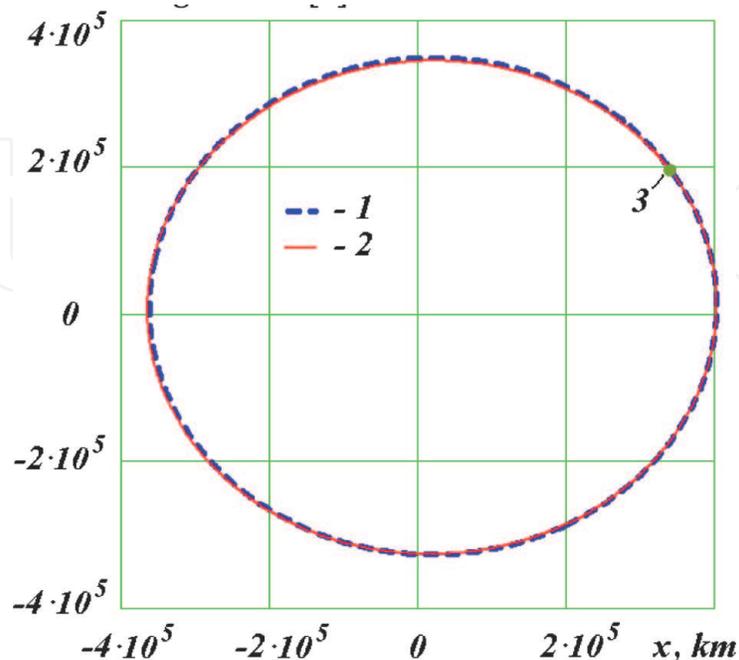
Figure 7.
Comparison of the projections of the Moon's orbit onto the equatorial plane xy *calculated in two ways: (1) based on the results of numerical integration, by the Galactica program, of the differential equations of motion of Solar-system bodies; (2) according to the mathematical model of the Moon's motion; and (3) the starting point of the orbit at the moment* T = 0.

the interval of 113.4 years, the average values were designated as P_m, T_S, and $T_{\varphi p}$, respectively. Their magnitudes in sidereal years are given in **Table 4**. Astronomy considers different months with durations expressed in days. The sidereal month with a period P_{msid} is specified relative to fixed stars. The synodic month with a period P_{msyn} is specified in relation to the Earth. The sidereal orbital period of the Earth relative to stars is P_{Esid} = 365.25636042 days. Therefore, the angular velocity of the Moon in its orbit around the Earth relative to it is equal to the difference between the angular orbital velocities of the Moon and the Earth in relation to stars. Therefore, the duration of the synodic month is

$$P_{msyn} = \frac{P_{Esid} \cdot P_m}{P_{Esid} - P_m} \tag{18}$$

where the period P_m = 7.479277 · 10^{-2} sidereal years expressed in days is equal 27.318536.

The period P_{mano} of the anomalistic month is specified in relation to the Moon's perigee or its apogee. The period of motion of the Moon's perigee relative to stars is denoted as $T_{\varphi p}$. Therefore, the period of the anomalous month is

$$P_{mano} = \frac{T_{\varphi p} \cdot P_m}{T_{\varphi p} - P_m} \tag{19}$$

where the period $T_{\varphi p}$ is expressed in days.

The Draconic month with a period P_{mdra} is specified in relation to the Moon's ascending node. The position of the ascending node γ_{Mo} is specified by the angle $\varphi_{\Omega Mo}$ (**Figure 1**), and its motion relative to fixed stars occurs with the precession period T_S of the Moon's orbital axis $\vec{S}_{Mo}$. Therefore, the draconic-month period is

$$P_{mdra} = \frac{T_S \cdot P_m}{T_S - P_m} \tag{20}$$

where the period T_S is expressed in days.

A tropical month with a period P_{mtro} is defined in relation to the Earth's moving equator AA' in **Figure 1**. The moving equator, as well as the Earth's axis of rotation, precess relative to fixed stars with a period of P_{prEax} = −25738 sidereal years [1]. Therefore, the period of the tropical month will be:

Method	P_m	T_S	$T_{\varphi p}$	P_{msid}
		Siderial years		**Days**
Calculation	7.479277•10^{-2}	−18.60062	8.852804	27.318536
Observation	—	—	—	27.321662
Relative difference, δ	—	—	—	−1.14•10^{-4}
Method	P_{msyn}	P_{mano}	P_{mdra}	P_{mtro}
		Days		
Calculation	29.526938	27.551303	27.209129	27.315564
Observation	29.530589	27.554550	27.21221	27.321582
Relative difference, δ	−1.24•10^{-4}	−1.18•10^{-4}	−1.13•10^{-4}	−1.14•10^{-4}

Table 4.
Comparison of calculated and observed average durations of various months: sidereal P$_{msid}$, *synoptic* P$_{msyn}$, *anomalistic* P$_{mano}$, *and draconian* P$_{mdra}$.

$$P_{mtro} = \frac{P_{prEax} \cdot P_m}{P_{prEax} - P_m} \qquad (21)$$

where the period P_{prEax} is expressed in days.

These periods, as calculated by Eqs. (18)–(21) and as evaluated from the observations of [10] are summarized in **Table 4**. The relative difference between the calculated and observed periods is expressed in terms of a parameter δ defined similarly to Eq. (17). As it is seen, the largest value of δ is $1.24 \cdot 10^{-4}$. The main contribution to this difference is made by the sidereal period P_m of Moon's orbital revolution. If we use the observed value of P_m = 27.321662 days, then the δ-values will decrease by two-five orders of magnitude.

As it is seen from **Figure 2**, the Moon's orbital period P experiences oscillations with relative amplitudes ΔP_{A1} and ΔP_{A2}, which in total make up 0.0053 part of the period P. In addition, from **Figure 6** it is seen that over time intervals of tens of thousands and more years there exist oscillations of the average period P_m with a relative amplitude of the order of $2 \cdot 10^{-4}$. For oscillating quantities, their average values depend on the interval over which the averaging is performed. The value of P_m given in **Table 4** was obtained by an averaging performed over an interval of 113.4 years, and the value of the sidereal period in astronomy has an averaging interval of about 2 thousand years. This seems to be the main reason for the difference between the calculated and observational data with a relative value of the order of $1 \cdot 10^{-4}$.

3. Prospects for on-Moon research

3.1 Problems, and their content and structure

There are various proposals for research to be carried out on the Moon. Some of those proposals may prove useful, while others, not [11]. The Moon near the Earth is the only body close to it. Therefore, not counting the Earth, the Moon is the only body that can be used for the study and exploration of outer space. It seems that such activities should be carried out along three lines. It is necessary to study the Earth, the Sun, and outer space from the Moon. For this purpose, an Earth Service should be established on the visible side of the Moon, and a Space Service, on its opposite side. Solar exploration will be additionally performed by both Services.

The mission of the Earth Service is to continuously monitor and analyze all processes and phenomena that occur on the Earth. Observations should be carried out using optical means in all ranges of the spectrum. In addition, other available methods known in astronomy for measuring the physical characteristics of the Earth, such as the methods of radio astronomy, γ-astronomy, methods for measuring the magnetic properties of the Earth's surface, and others, should be used. The results of such measurements will provide a better insight into the processes taking place on the Earth. As a result, it will become possible to improve methods for the long-term forecasting of weather and such catastrophic events as tropical cyclones, hurricanes, typhoons, etc. Continuous observations of the Earth will provide reliable data on many events and processes occurring on the planet: the state of ice conditions in the southern and northern oceans, the dynamics of snow cover, various seasonal changes of the Earth's surface, fire hazard of territories, volcanic eruptions, man-made accidents and disasters, the fall of large meteorites, as well as various military actions on a global scale.

All this will contribute to a safer and more stable habitation of humans on the Earth.

The Solar Service, located on both opposite hemispheres, will allow the observation of processes on the Sun in an almost continuous mode. Solar flares affect the dynamics of the Earth's atmosphere, and they presently cause many dangerous atmospheric phenomena [12]. The Sun's activity, manifested in the number of sunspots, varies periodically. Such periods correlate with the periods of the Sun's movement around the center of mass of the Solar-system [12, 13]. Their duration is 22 years with two sub-periods each lasting 11 years. In addition, there are also large periods lasting hundreds of years. Possibly, those fluctuations of the Sun's activity cause the short-period variations of the Earth's climate [13].

The study of solar processes will allow a more detailed understanding of the processes occurring on stars. The two Solar Services will host the equipment used for studying the Sun and stars from the Earth. The effectiveness of the use of this equipment on the Moon is expected to be much higher, as there is no cloudiness and no atmosphere there. Due to the small force of gravity, structures cumbersome on the Earth will appear weighing much less on the Moon.

The Space Service is the most important part of human activity on the Moon. The importance and relevance of its tasks to the solution of many challenging problems will permanently grow in time. At an early stage, this service will carry out all studies currently being carried out on the Earth with the help of Earth's satellites. As this service evolves, these tasks will be supplemented with new ones that cannot be accomplished with the help of satellites. One of such tasks is the communication with spacecraft sent into deep space. The absence of atmosphere and intrinsic magnetic field on the Moon will make it possible to carry out such connections in a more stable manner.

What divisions should be included in these two services? Each service should consist of the following three departments: (1) Research Department; (2) Engineering Department; and (3) Greenhouse Department.

The task of the Research Department is to carry out works on the study of the Earth, the Sun, and space. The task of the Engineering and Technical Departments is to create the material base of the service and ensure its functioning. The task of the Greenhouse Department is to support life on the Moon, provide food for inhabitants, and to ensure life in all structures of the greenhouse economy.

At the first stage, the tasks of the Greenhouse Department will come as the main ones, since the human civilization presently has no experience in supporting life in extraterrestrial conditions. Work on the Earth and artificial earth satellites to create life in artificial conditions should begin in advance. Some experience in this area already exists. It is necessary to study this experience and formulate a research program for the creation of various life elements in extra-terrestrial conditions in relation to the Moon. After accomplishing this work, we can initiate the development of a greenhouse project on the Moon.

Until the full-fledged functioning of the greenhouse economy begins on the Moon, research and engineering works will mainly be carried out with the help of automatic machines and mechanisms controlled from the Earth.

3.2 Transportation on the Moon

For moving on the Moon, it will be necessary to create walking and running vehicles. Animals on the Earth, two- and four-legged, can move at a decent speed comparable with the speed of wheeled vehicles. But an animal can move at this speed in off-road conditions. When moving, the animal observes its path and puts its foot on the ground taking into account all the circumstances arising at the point of contact with the ground. Modern means of observation, monitoring, and control make it possible to create a mechanical leg of a vehicle that will function no worse

than the leg of the fastest animal. In further development, a vehicle with mechanical legs will reach in off-road conditions the speed of a wheeled vehicle on a good road.

Such vehicles with mechanical legs can be supplemented with mechanical arms or some legs can be provided with the function of arms. Mechanical arms will help the vehicle to extricate itself from emergency situations: when overturning, when driving in dangerous areas, etc. Control algorithms shall be developed for different situations and with time the reliability of such vehicles will approach 100%.

When driving on established routes, a vehicle with mechanical arms can clear the most disturbing obstacles out of the way. In this way, paths and roads for this transport will be created, along which the speed of movement will be increased.

Such vehicles, equipped with navigation aids, will be able to move with or without man. All works related to the delivery of goods will be executed without people. This will greatly simplify, and reduce the cost of, moving goods, since there will be no need in using life support systems for people.

Long-distance movements, for example, those between the Earth and the Space Services, will be performed using jet engines along ballistic trajectories. In jet engines on the Earth, fuel burns in an oxidizing agent, the combustion products acquire a high speed, and the jet stream propels the vehicle, for example, a space-craft. In lunar jet engines, lunar sand and dust will be used as the jet substance. The jet vehicle must possess the energy required to impart the speed of the jet stream to this material. This energy can be the electrical energy stored in batteries. The batteries will be charged by solar panels during the lunar day.

The acceleration of the substance can be carried out electrically. For example, a charge of one sign can be imparted to a bulk material, which then enters an inter-electrode space with a high voltage to undergo acceleration. In the mechanical method, the bulk material is fed to a rotating device to acquire the required speed. In this case, in order to prevent the vehicle from rotating, it is necessary to have paired devices rotating in different directions.

As the bulk material, lunar regolith can be used, which, apparently, includes terrestrial analogs in terms of its granulometric composition such as dust, powder, sand, and sandy loam.

The issue of obtaining and storing energy is a special problem that requires careful study. Apparently, in the non-polar regions of the Moon, solar energy will be sufficient. Solar panels can provide electricity that needs to be stored for the Moon night. For heating during the night and for cooling during the day, respectively heat and cold accumulators must be used. Electricity can also be generated based on the temperature difference between the lunar surface and the constant-temperature layer beneath it. This temperature difference exists both during the day and at night. Apparently, Stirling engines can be used here for doing work and for generating electricity.

3.3 Materials and substances

For the creation of the Earth and Space Services, various materials and substances are needed. Consider what is required for supporting life on the Moon. Greenhouses will need soil, water, and air to function. Soil samples can be taken from the Earth. When plants settle on them, the soil can be mixed with lunar soil, with its amount being gradually increased. Apparently, not every lunar soil is suitable for these purposes. Therefore, a lot of work needs to be done to study the lunar soil, prepare and collect the required composition, and deliver it to greenhouses.

Where can we get water? During lunar days, the Moon's surface gets heated, and the water boiled away and evaporated. It is necessary to study the distribution of

temperature over the lunar surface. Somewhere closer to the poles, a negative temperature can be found. It might be possible to find ice there.

In equatorial and middle latitudes, the temperature of the lunar surface varies from hundreds of Celsius degrees during the day to hundreds of degrees below zero at night. But with depth, the layer of variable temperature must vanish, and a constant temperature must establish. How low is this temperature? If the temperature is negative, then there may be ice found at this depth.

Thus, in order to find water, one has to carry out temperature studies of the Moon, both in-depth and over the surface.

Where can we get air? On the Earth, air contains 80% of nitrogen and 20% of oxygen. There are also small amounts of other gases. Apparently, many of them are not necessary.

There are no ready air and component gases (nitrogen and oxygen) on the Moon. Therefore, they must be obtained from substances available on the Moon. It is necessary to study the composition of lunar rocks. Then, people on the Earth must develop technologies for the extraction of nitrogen and oxygen from these rocks. Subsequently, the composition of the artificial air can be optimized with the help of plants and algae. Among them are those that give off oxygen as well as other gases.

For the construction of a greenhouse, structural materials, metals, and various substances are needed. It is impossible to get them from the Earth. From the Earth, it will be necessary to transport finished products, complex instruments and tools, machines, and similar products, which are impossible to manufacture on the Moon. All necessary materials and substances must be extracted from minerals available on the Moon. That is why the Moon's geology must be well studied. On its basis, processes on the transformation of lunar minerals into necessary materials and substances should be developed on the Earth.

3.4 Safety of buildings on the Moon

Buildings on the Moon will require a lot of spent effort, money, and time. Therefore, they must be durable with a service life amounting to hundreds of years. In this regard, it will be necessary for people on the Moon to protect themselves from natural disasters. This can be soil creeps on slopes, rockfalls, meteorite falls, etc. Some of such processes and events can pose no real threat. That is why, before the start of construction, it will be necessary to perform a study of possible risks and their occurrence probabilities. As for the meteorite danger, its reality is beyond doubt, since the entire surface of the Moon, like that of all celestial bodies, is dotted with meteorite craters. Therefore, this threat must be treated with close attention. Apparently, it is necessary to conduct experimental observations on the probability, composition, and characteristics of meteorites falling onto the Moon. For this purpose, it is possible to spread a screen on the Moon's surface with means for observing and controlling the fall of meteorites. Information from such devices must be transmitted to Earth. Observations should be made over several years. They will allow scientists to obtain data on meteorite hazards, which is necessary for the design of buildings. There should be two such sites in the places of proposed construction: one on the visible side of the Moon, and the other on the opposite side.

Over the long service life of structures, there will always be a danger of being hit by large meteorites. Therefore, a vitally important part of the greenhouse must be created below the Moon's surface. Apparently, the best option would be the creation of each service near a rock hill. The greenhouse farm will be located outside the hill, with its all vitally important systems being hidden in hollow rooms inside the hill. The top of the hill will provide the reliable protection of such systems from relatively large meteorites. The greenhouse should be made sectioned. Then, in the

event of a depressurization having occurred at some section as a result of a meteorite hit, the remaining sections will automatically be cut off from the damaged section and continue to function.

3.5 The relations between humans in their activities on the Moon

Services on the Moon will be created in the interests of all mankind. However, there are states on Earth the relations among which cannot be called friendly. Mutual threats are possible and wars to destroy each other are not ruled out. This situation may not radically change in the next hundreds of years. Therefore, principles to govern the relations between people of the Earth during their activities on the Moon must be formulated. Based on the conditions necessary for the successful functioning of two services on the Moon, let us try to formulate some of those principles.

First, each state has the right to take part in the creation and functioning of these services, and it will share the results obtained.

Secondly, since there are two services, it makes sense to form two groups of states, one being responsible for the service on the visible side of the Moon, and the other, on its backside.

Thirdly, having obtained permission, the representatives of one group of states will have the right to visit the territory of the service shared by the second group of states.

Fourth, each group of states shall share its achievements and results with the members of the other group at no cost.

Fifth, unfriendly and hostile relations among states on the Earth shall not be practiced by representatives of such states on the Moon.

Those who call to violate this principle will be subject to capital punishment with no statute of limitations.

Mankind already has experience of such cooperation gained in the study of Antarctica, in the Apollo-Soyuz project, and in the activities at the International Space Station. This experience can be considered successful. For cooperation on the Moon, the accumulated experience shall be widely applied.

3.6 Work sequence

Moon exploration began 50 years ago by the Soviet Union and the United States. Other countries now take part in it. This activity will be continued by different countries in the future. For making those fragmented studies fruitful, it is necessary to set common goals and formulate certain tasks. Then all the studies will add to our common knowledge of the Moon, which subsequently will allow these goals to be achieved.

Therefore, it is necessary to conduct an international discussion of the problem of Moon exploration by all interested parties. The result of this discussion should be the establishment of an International Committee for Moon Exploration. The first task of this committee shall be the development of a preliminary project on the prospects for Moon exploration.

In this project, all the goals and objectives discussed above will be concretized. This will allow different countries to unite their efforts. The International Committee will have the task of coordinating these studies, analyzing and summarizing their results, and setting further tasks.

This work will contribute to the rapprochement of the individual parties, uniting them in the implementation of large projects. This cooperation will further lead to

the consolidation of collaboration teams necessary for the creation of Earth and Space services.

One of these preliminary tasks is the creation of a Moon satellite. The satellite is needed as an intermediate station for flights from Earth to the Moon and back. In addition, a satellite is needed to connect the Space Service with the Earth, and the Earth and Space services with each other.

Further development of the International Committee for Moon Exploration will turn it into main mankind's organization on the exploration of the Moon and lunar works.

3.7 Possible missions to be performed using Moon services

When mankind starts establishing services on the Moon, the task may be set to provide the Moon with a long-term satellite. Previously, we have performed trajectory calculations for transforming the Apophis and 1950DA asteroids into Earth's satellites [14]. The task here is to choose an asteroid suitable for making it a Moon satellite. Apparently, the orbit of such a satellite should be circular or having a small eccentricity and a semi-major axis about 5000 km long. That is, the spacing between such an asteroid and the Moon should be equal to the above distance. The satellite's orbit must lie in the Moon's orbital plane. Such a satellite will increase the reliability of movements between the Earth and the Moon.

In astronomy, various methods are used to determine the distance from the Earth to astronomical objects. The most reliable one is the triangulation method, in which the angles of observation of a star from opposite points in the Earth's orbit are measured. The angles can be determined from the displacement of a star over the celestial sphere against the background of more distant stars. In this way, one can measure the distance to objects located at a distance of 20 parsecs (pc). In this case, the base distance is the semi-axis of the Earth's orbit a. On increasing the base length, the range of measured distances will increase in proportion to this length.

One can increase the base by placing one of the observation points on a spacecraft launched from the Earth along a hyperbolic orbit. The location of the star, observed on the spacecraft at some distance r from the Earth and communicated to it, will make it possible to determine the distance to stars located at typical distances greater than 20 pc by a factor of r/a.

We assume that a spacecraft is launched at point A in **Figure 8** in the Earth's orbital plane in the direction of Earth's orbital motion. Suppose, for instance, that the speed of the spacecraft relative to the Earth is 20 km/sec, and its speed relative to the Sun is 50 km/sec. At this speed, the spacecraft moves in a hyperbolic orbit, with its speed at infinity being v_∞ = 28 km/s, i.e. the spacecraft leaves the Solar system at this speed. Six months later, a similar spacecraft is directed at point B in the opposite direction.
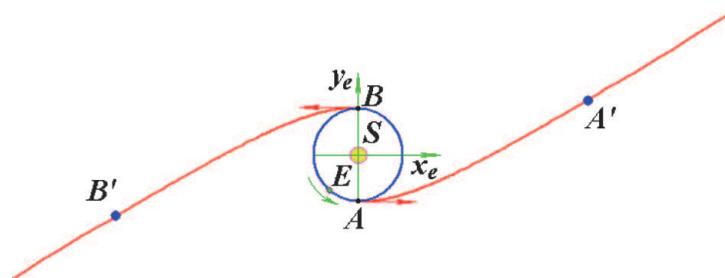
Figure 8.
Trajectories of a triangulation spacecraft for measuring distances to stars: S – *the Sun;* E – *the Earth;* A *and* B – *the launch points of spacecrafts* A' *and* B', *respectively;* $y_e x_e$ – *the plane of the heliocentric ecliptic coordinate system* $x_e y_e z_e$ *for the epoch 2000.0.*

Parameters	Parameter values									
T, years	1.03	2.12	3.08	4.05	5.02	10.1	15.1	20.4	25.4	30.4
r, AU	7	14.4	20	26	32	63	93	125	155	185
D, pc	140	280	400	520	640	1260	1860	2500	3100	3700

Table 5.
The distance D *to stars as determined by triangulation spacecraft, depending on the time of their movement* T *over a distance* r *from the Sun.*

The views of the starry sky seen from the spacecraft in the direction of the z_e-axis and in the opposite direction shall be sent to the Earth at certain time intervals. The view seen from the spacecraft launched at point A can be compared with the view of the starry sky seen from spacecraft B located at the same distance r. This will permit the measurement of distances to objects of $D = 20r/a$ (in parsecs). **Table 5** shows the time of observation T, the distance r from the Sun in astronomical units, and the distance D to astronomical objects, which will be determined using triangulation satellites. After a year of motion, we will be able to reliably know the distance to stars located at a distance $D = 140$ pc; after ten years, at a distance of 1260 pc; and after 30 years, 3700 pc. It should be noted that at a distance of 20 pc, the distance from the Earth will be determined with an error of 20%. Therefore, with an increase of the distance r to the spacecraft, it will become possible to refine distances to objects located at distances smaller than the value D indicated in **Table 5**.

Range measurements are possible for those distances r, up to which the exchange with data between the Space Service on the Moon and the triangulation spacecraft is possible.

Distance D to astronomical objects is the basic parameter in astronomy. The sizes of an object, its speed, physical characteristics, and in some cases, it is very physical nature depending on the distance. That is why, in order to be confident in its knowledge of deep space, mankind will always be faced with the task of refining distances to space objects.

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