Numerical Investigation of the Mars Orbit Evolution in the Time Interval of Hundred Million Years

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Abstract. *We discuss here our results devoted to the problem of dynamical evolution of the Solar System during large time intervals, in the order of hundred million years.*

Our interest to the problem is induced first of all by a very complicated science-philosophical problem of the origin, evolution and stability of the Solar system. This interest is dictated also by the climate investigation at our planet and its changes under the influence of the Earth orbital and rotational motion evolution in the framework of so-called Astronomical theory of the boulder periods [1].

According to the classical dynamics motion of bodies in some inertial frame of reference is determined by the following system of $6n$ differential equations [2]

$$\frac{d^2\vec{r}_i}{dt^2} = -G \sum_{k\neq i}^{n} \frac{m_k \vec{r}_{ik}}{r_{ik}^3}, \quad (i = 1, 2, \dots, n), \tag{1}$$

where $\vec{r}_i$ is a radius-vector of the body m_i with respect to the center of mass of the system (for example, with respect to the barycenter of the Solar system).

For $n = 11$ (ten major planets, the Sun and the Moon) equations (1) are the system of differential equations describing various motions in the framework of the chosen model for the Solar system.

We have done a computational experiment using two variants of the initial conditions for integrating the system (1). In the first and the second cases we used the ephemerids DE 19 [3] and DE406/LE406 [4] of Laboratory of Jet Propulsion of the USA.

To integrate the system (1) we have developed an algorithm and its implementation in the computer program named "Galactica" [5]. The essence of this algorithm is that the value of the calculated function at the instant of time $t = t_0 + \Delta t$ is calculated by means of the Taylor series

$$x = x_0 + \sum_{k=1}^{K} \frac{1}{k!}\, x_0^{(k)} (\Delta t)^k , \tag{2}$$

where $x_0^{(k)}$ is the kth order derivative at the instant of time t_0, and an integer K is the derivative highest order.

A value of the velocity x' (the first derivative of the coordinate x) is determined in a similar way while an acceleration x'' is calculated according to the formula (2). The higher order derivatives $x_0^{(k)}$ we have determined by means of direct differentiation of the right-hand sides of the equations (1) in analytical form.

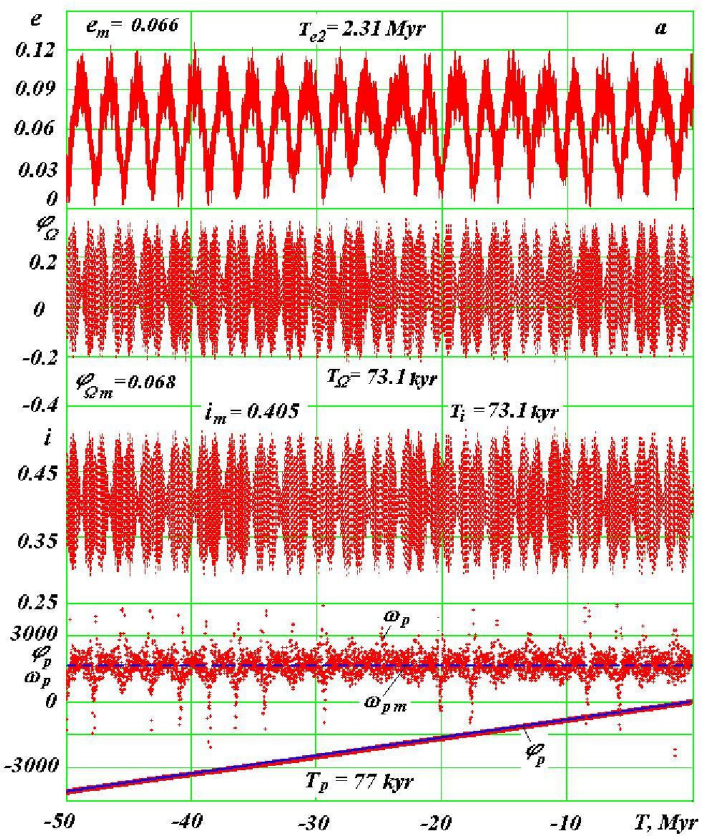

Figure 1: The Mars orbit evolution during 50 million years: T is a time measured in million years; time interval between the neighboring points is equal to 10 thousand years; e is an eccentricity, i is an inclination of the orbit plane with respect to equator plane in epoch 1950.0 in radians; φ_Ω is an angular position of the orbit ascending node with respect to Ox axis in epoch 1950.0 in radians; φ_p is an angular position of perihelion in the orbital plane with respect to the ascending node in radians; ω_p is an angular velocity of the perihelion rotation measured in seconds of arc per hundred years in the time interval of 20 thousand years; average value of the angular velocity during 50 million years $\omega_{pm} = 1687$ seconds of arc per hundred years.

Figure 1 shows evolution of parameters of the Mars orbit during the time interval of 50 million years. One can distinctly see the second period of the eccentricity change being equal to $T_{e2} = 2,31$ million years when the lowest $e = 0.0014$ and the highest $e = 0.126$ values of the eccentricity are observed. Oscillations of the inclination angle i occur within the bounds of 0.288 and 0.521 radians and their interval is 13.37 deg. Perihelion moves in the direction of the Mars orbiting the Sun with an average for 50 million years period T_p. Angular velocity ω_p of the perihelion rotation oscillates about its average value ω_{pm}. Comparison with a graph of the eccentricity shows that return motion of the perihelion occur when the orbit eccentricity is close to zero. The shown graphs demonstrate that amplitude and period of the Mars orbit oscillations are stable, i.e., the Mars orbit motion is stationary.

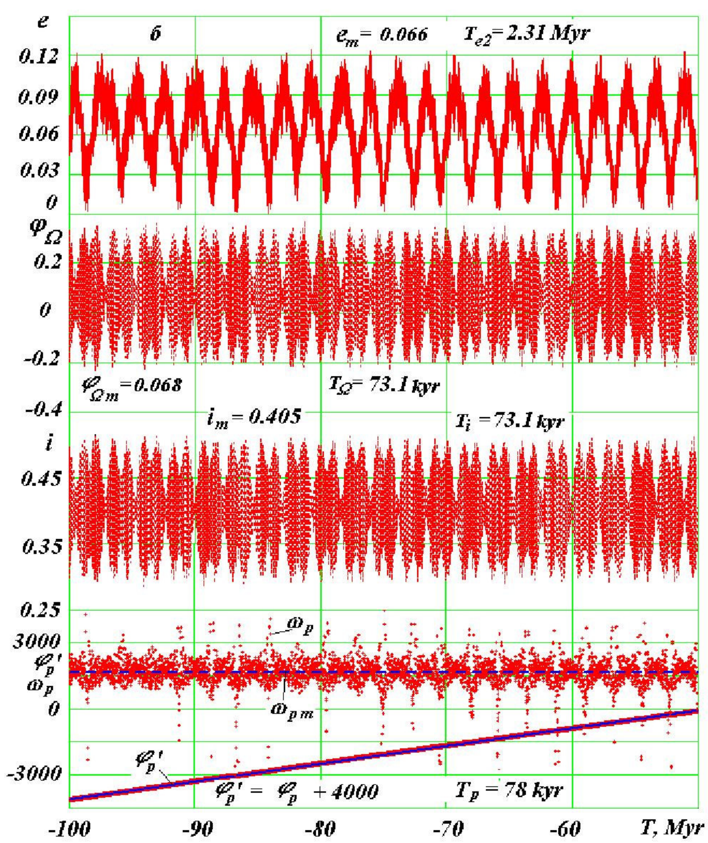

Figure 2: The Mars orbit evolution in time interval $T \in (-100, -50)$ million years.

Fig. 2 shows evolution of the Mars orbit parameters in the time interval $T \in (-100, -50)$ million years. Comparing these graphs with those of Fig. 1, one can see that character of the orbit parameters evolution during two successive time intervals in 50 million years doesn't change. Periods of oscillations of the eccentricity e, the ascending node of the orbit φ_Ω and the angle i of its plane inclination with respect to the stationary plane of the equator as well as their amplitudes and period of revolution T_p remain the same. Similar results have been obtained for the orbits of other planets during the same time interval in 100 million years. This is evidence of a stable character of the planets motion in the Solar system.

During the last three hundred years different aspects of the Solar system stability were subjects of investigation of many outstanding scientists (I. Newton, L. Euler, J. Lagrange, P. Laplace, K. Gauss, A. Poincare, A.M. Liapunov, O. Yu. Shmidt, A.N. Kolmogorov, N.D. Moiseev, G.N. Duboshin and others) but in spite of their efforts this fundamental problem has not been got a complete solution until now.

Considering motion of any celestial body in "phase space" with "slow" and "fast" variables, we have shown that all slow variables (major semiaxis, eccentricity, inclination, longitude of the ascending node and angular argument of perihelion) behave as conditionally periodic functions what corresponds to the known mathematical results on proving the averaging methods (N.N. Bogolyubov, Yu.A. Mitropolski, A.M. Samoylenko, E.A. Grebenikov, Yu.A. Ryabov and others).

Numerical investigations we made show that during the last 100 million years the Solar system is stable and there is no any tendency to some catastrophic changes in the geometry and dynamics of the Solar system.

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