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THE MATHEMATICAL MODELS OF THE ROTATED LAYERS

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Poster section

The weighted layers of particles are applied in the chemical technology and power engineering. The velocity of weighing of a flow is limited by weight force of a particle. In a rotated layer the weighing is executed against centrifugal forces, which there can be on the order more force of weight. Therefore the productivity and efficiency of such weighted layers is growing. The rotated layers are too used in other applications, for example, with rotary drilling a rotated layer of dirt accompanies. Therefore the considered problem has rather important significance.

The rotated layers can be realized in a rotated chamber (centrifuge) and in a stationary vortex chamber. As a result of theoretical and experimental researches [1] we have come to a conclusion, that for creation of a rotated layer it is necessary to take into account three problems: 1) structure of flow of bearing medium in the vortex chamber; 2) the influences of the particles to this flow; 3) interactions of particles with changed flows. We are considering two kinds of a layer: 1) rotated layer, which are touching with a cylindrical wall of the chamber; 2) the weighted rotated layer.

Rotated layer.

At this case the centrifugal force, stipulated by tangential velocity of the particle v_p, press it to the cylindrical wall of the chamber, than it is created the braking of the layer and flow. We consider a rotated layer in the cylindrical vortex chamber with the radius R_c and height L. The bearing media comes under a angle of the inclination to radius ψ_i through inlets with the common are f_i in the cylindrical wall of the chamber, which is the swirler. It leaves through the outlet of the radius R_1 in the end wall. The moment of forces, stipulated by outflow of gas from swirler, is inlet moment and equal

$$\Omega_i = \rho\, Q v_i R_{\tilde{n}} = G\, \Gamma_c \qquad (1)$$

where Q, G is volumetric and mass flow rate of gas; v_i is tangential velocity in the channels of swirler; $\Gamma_c = v_c R_c$ is the circulation on the periphery of the chamber; v_c is tangential velocity of a gas on a periphery of the chamber. The moment of forces is on the one hand enclosed to swirler, and with other creates an angular momentum of moving flow Ω_i. The moment of friction Ω_f of a layer about cylindrical swirler counteracts to the inlet moment. The other part of moment Ω_i creates a stream of media momentum Ω, of passing through the layer, i.e.

$$\Omega_i - \Omega_f = \Omega. \qquad (2)$$

If f_i is the friction coefficient of a layer about a surface of swirler, and F is the pressed force of particles to it, such particle will create moment of friction $\Omega_l = F R_c f_f$. The pressed force of the particle in a radial direction $F = m_p * du_p/dt$, where we used the radial acceleration of the particles du_p/dt at the laminar regime of the flow [1]. Considering a particle on a periphery of the chamber at radial velocity u_p, we receive the force expression

$$F=-\frac{m_p u_c^2 S_p}{R_c}(1-K_S^2)\,, \tag{3}$$

where

$$S_p=\frac{3\pi\mu d_a R_c}{m_p(-u_c)}\;; \tag{4}$$

$$K_S=K\sqrt{\frac{1-\overline{\rho}}{S_p}}\;; \tag{5}$$

K_S is separation coefficient, which was developed in [1]; d_a- is aerodynamic diameter of the particles; $\overline{\rho}=\rho/\rho_p$ is the relative density; $u_c = -Q/(2\pi R_c L)$ is average to flow rate radial velocity of a gas; ρ_p, m_p is density and mass of a particle; $K = v_c/(-u_c)$ is spiral degree of a flow in the vortex chamber. Here coefficient K_S depends on the spiral degree K_L in a loaded vortex chamber. From expression (3) follows, if the separation coefficient is less one the force is negative, i.e. it is directed to a centre of chamber. At $K_S > 1$ the force is positive and presses a particle to the cylindrical surface of the chamber. Then the moment of friction, created by one particle is defined so

$$\Omega_1=-m_p u_c^2 S_p(1-K_S^2)f_f\;. \tag{6}$$

Summarising (6) on all particles the moment of friction can be written

$$\Omega_f=-u_c^2 f_f\sum_{d_{cr}}^{d_{max}} d_a\left[\frac{3\pi\mu R_c}{(-u_c)}-\frac{\pi d^2\rho_p K_L^2}{6}(1-\overline{\rho})\right], \tag{7}$$

where d_{cr} is defined from a condition $K_S = 1$ and at laminar regime will be such:

$$d_{cr}=\sqrt{\frac{18\nu\overline{\rho}R_c}{K_L^2(1-\overline{\rho})(-u_c)}} \tag{8}$$

Here d_{max} is diameter of the heaviest particle in the layer.

As at $d_a > d_{cr}$ the first term in (7) is considerably less then second one and at summation it can be neglected, we have a moment of friction in such form

$$\Omega_f\approx v_{pc}^2(d_a/d_s)f_f(1-\overline{\rho})\sum_{d_{cr}}^{d_{max}} m_p\;\;, \tag{9}$$

where d_s is diameter of a spherical particle with volume of particle V_p.

In (8) the spiral degree $K_L = v_{pc}/u_c$ depends on tangential velocity v_{pc} of a gas and layer on the periphery of the chamber. Here $v_{pc} = v_c\,(s)^{0.5}$, where s is the parameter of braking of flow by the layer. The parameters s is entered in [1] on changing of the pressure in the loaded vortex chamber. The particles in the layer are considered, the sizes of which are distributed on normally - logarithmic law

$$D=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\tau}\exp(-0.5\tau^2)d\tau\,, \tag{10}$$

where

$$\tau = \frac{\lg(d / d_{50})}{\lg \sigma}. \tag{11}$$

Passing from summation in (9) to integration of the particles mass

$$dM = M \frac{dD}{dd} dd$$

where M is mass of all particles in the layer ,we finally receive the moment of friction of the layer

$$\Omega_f \approx q(d_a/d_s) M f_f (1-\overline{\rho})[1-D(\tau_{kr})] v^2{}_{pc} , \tag{12}$$

where

$$\tau_{cr} = \frac{\lg(d_{cr} / d_{50})}{\lg \sigma}; \tag{13}$$

q is geometrical coefficient of particle.

In the steady flow regime the tangential velocity of media, passed through a rotated layer, is equal tangential of velocity of the layer v_p. Therefore the stream of the media momentum is equal

$$\Omega = \rho Q v_{pc} R_c..$$

After substitution of moments in (2) and fulfilment of transformations we find the equation for the parameter of braking:

$$\frac{1}{s} = \frac{M(1-\overline{\rho})[1-D(\tau_{kr})] f_f d_a \sin \psi_i}{\rho \cdot R_c f_i d_s} + \frac{1}{\sqrt{s}}. \tag{14}$$

We executed the experiments [2] on creation of a layer of particles in the vortex chamber with R_c =80 *mm* and L = 200 *mm*. As a result of the researches we received the experimental dependence for parameter of braking, which is similar to equation (14). With its account the generalized experimental dependence for parameter of braking can be written so:

$$\frac{1}{s} \approx \overline{M}(1-\overline{\rho})[1-D(\tau_{cr})] f_f + 1, \tag{15}$$

where $\overline{M} = M d_a \sin \psi_i / \rho R_c f_i d_c$ *is* relative mass of the layer.

In the work [1] it is shown, that the maximum particle diameter in the layer is defined by expression

$$K_{S\max} < \frac{8.8}{\sqrt{s}} \tag{16}$$

and the minimum particle diameter is defined

$$K_{S\min} > \frac{1}{\sqrt{s}} \left(\frac{R_1}{R_c} \right)^n, \tag{17}$$

where $n = [1 + 1*10^{-4} K^2 / (L/(2R_c))]^{-0.5}$.

The expression (15) permits to define the parameter of braking and to close the system of equations (16) - (17) on the mechanic of a rotated layer. At calculation s from (15) it is necessary to take into account, that d_{cr} through K_L depends on s. Therefore the parameter s is calculated by a method consecutive approximation. Thus the created mathematical model of a rotated layer consists of a closed system of algebraic equations. It has be developed the computer program "Rotlayer". At giving geometry of the chamber, physical properties of medium and particles, flow rate parameters, one is determined the mass of particles in the layer, which are touched with a wall and their sizes; the masses of the particles, which are weighted and are thrown out from the chamber and their sizes.

Weighted layer of particles in the vortex chamber.

At certain parameters all particles will be in a weighted condition, thus the weighted rotated layer will be formed. The direct braking of particles about a cylindrical wall of the chamber is away in this case. For more exact calculation of the weighted rotated layer it is necessary to take into account braking of the layer about the end walls of the chamber. If a critical diameter d_{cr} is more the maximum diameter d_{max} for the particles being present in the layer, all particles will be weighted. At this, according to (13), $\tau_{cr} >> 1$ and $D(\tau_{cr}) \approx 1$. Therefore from (15) parameter $s = 1$, i.e. the completely weighted layer will not be brake the flow about the cylindrical surface of the chamber. With decrease of such braking the significance of braking of the layers about end walls grows. This kind of braking plays a more essential role for small-sized particles weighted in liquid flow, in a chamber with hyperbolic walls [3] and gas-liquid vortex chamber [4]. Pursuant to [5] for a completely weighted layer we shall take into account the braking of a flow about end walls. It is considered the homogenical weighted cylindrical layer, which has length L, the external radius R_e and internal R_i , and mass of particles in the layer M. The average density of a layer can be written:

$$\rho_l = \frac{\rho \quad V_m + \rho_p V_{p\Sigma}}{V} , \tag{18}$$

where V is volume of the layer; V_m is the volume, occupied by media; $V_{p\Sigma}$ is the volume of all particles.

We shall consider interaction of a ring weighted layer in radius r and width $\mathbf{d}r$ with end walls of the chamber. The force of action of the biphase flow is defined by dynamical pressure $C_f \ \rho_l \ (v_l)^2/2$, where C_f is a friction coefficient of the flow about the wall. Then the moment of forces of action of the ring layer on the two end walls is

$$d\Omega = 2C_f \frac{\rho_l v_l^2}{2} 2\pi r r dr = 2\pi \rho_l C_f \Gamma^2 dr , \tag{19}$$

where $\Gamma = v_l \, r$ is the circulation of flow, passing through layer; v_l is the tangential velocity of media and the layer particles.

This moment of forces results in decrease the stream of the medium momentum $\Omega = G\Gamma$ passing through the layer of $\mathbf{d}r$:

$$\mathbf{d}\Omega = G\mathbf{d}\Gamma. \tag{20}$$

Excepting $\mathbf{d}\Omega$ from (19) and (20), we receive the equation

$$G\mathbf{d}\Gamma = 2\pi C_f \Gamma^2 \rho_l \, dr,$$

after integration of which under a boundary condition $\Gamma(R_e) = \Gamma_{lce}$ on outside radius of the layer R_e we have

$$\Gamma = \frac{\Gamma_{lc}}{1 + 2\pi C_f \rho_l (R_e - r)\Gamma_{lc} / G} . \tag{21}$$

As outside radius of the layer R_e can be less R_c, the circulation will be constant in the area $R_c \le r \le R_e$. Therefore it is possible to record

$$\Gamma_{lc} = v_{lc} R_c = v_{lc} R_e , \tag{22}$$

where v_{lc} is tangential velocity of media on the periphery of loaded chamber; v_{le} is tangential velocity of the layer and media at radius R_e..

At the completely weighted layer the particle do not pressed to the cylindrical surface of the chamber and do not create a moment of layer friction Ω_f. However the rotated weighted layer by means of bearing media interacts with a cylindrical wall, and the interaction in this case becomes essential. Similarly (19) the moment of friction about the cylindrical surface we shall write in such form:

$$\Omega_c = \pi C_f \rho_l \Gamma_{lc}^2 L$$

This moment results in decrease of the initial stream of media momentum Ω_i according to (1) up to size of the stream momentum on layer periphery, i.e.

$$\Omega_i - \Omega_c = G \Gamma_{lc}$$

From here

$$\Gamma_{lc} = \frac{\sqrt{4\pi C_f \rho_l \Gamma_c L / G + 1} - 1}{2\pi C_f \rho_l L / G} , \tag{23}$$

where $\Gamma_c = v_c R_c$; v_c is tangential velocity on the periphery of the unloaded vortex chamber.

The deduced equations depend on the friction coefficient. As a result of comparison of the theory and experiment the authors [5] are received $C_f = 0.003$, and for gas-liquid of a layer by analogy to a dispersal-ring flow in pipes authors [4] are offered $C_f = 0.005$. The such order of significance for the resistance coefficient follows from the Prandtl's formula [6]:

$$C_f = \frac{0.077}{\mathrm{Re}^{0.2}} \tag{24}$$

at $5*10^5 < Re < 10^7$, which generalizes the experiments on resistance of the smooth plate. For the vortex chamber $Re = v_c R_c / \nu$.

The system of algebraic equations is thus received, which determines the law of change of particle tangential velocity in the layer. It is solved by a method of consecutive approximate. At first the internal and outside radii of a layer are setting. Then the equilibrium orbits for the heaviest and least particles in the layer are calculated. The radii of orbits are accepted for a new layer borders and the process proceeds so long as the borders of a layer will not coincide with radii of orbits of these limiting particles. On this algorithm the computer program "Flulayer" is developed. Initial data are a chamber geometry, physical properties of medium and particles, disperse structure, including diameters of the heaviest and least particles and mass of a layer. As a result of the program work the velocity and diameter of particles on different radii of a layer are determined.

Developed mathematical models and their computer realizations permit us to solve the different problems with rotated layers of particles. Until today the process of the rotated layer creation was difficult experimental problem. The developed models permit to consider various variants of rotated layers, to select necessary dimensions vortex chambers or required parameters of rotated layers and to optimize them. These models can be used for study of process of rotary drilling and its intensification. They can be used at the study of the tornado with the purpose of determination of it's parameters and destructive properties.

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