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PALEOTEMPERATURES OF THE EARTH′S SURFACE. 2. DETERMINATION OF THE MIDLATITUDE NEAR-SURFACE HEAT-CAPACITY OF THE EARTH

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Two methods have been proposed for calculating the midlatitude near-surface heat-capacity of the Earth: 1) using the latitude variation of the Earth temperature, and 2) using the insolation of the Earth in the modern epoch and its variation with time. Using the obtained values of this heat capacity, profiles have been determined for the midlatitude paleotemperatures of the Earth in three extreme epochs in the past 50 thousand years, and the evolution of the paleotemperature in the past 200 thousand years has been calculated.

Keywords: *midlatitude temperature, insolation, heat capacity, paleotemperature, paleoclimate.*

Introduction. Consideration is given to the method of identifying paleotemperatures on the basis of data obtained in [1], and the results of calculations performed with its help are provided.

Dependence of the Near-Surface Temperature of the Earth on Its Insolation. It is known from physics that the increment of the heat quantity ΔQ supplied to a body results in a rise of its temperature by the value of

$$\Delta t = \Delta Q/(mC) , \tag{1}$$

where m and C are the mass and specific heat of the body. We use dependence (1) to investigate the processes of insolation of the Earth. The distribution of the annual heat supplied to the Earth is not uniform. We designate the specific heat capacity of the Earth surface (heat capacity per 1 m^2 of its surface) at the latitude φ as C_φ. In accordance with (1), we can consider that in the modern epoch, the temperature at the latitude $\varphi_2 = \varphi_1 + \Delta\varphi$ will change due to the change in the quantity of heat $Q_{t0}(\varphi_2)$ compared to the quantity of heat $Q_{t0}(\varphi_1)$ at the latitude φ_1 by the value

$$\Delta t_0 = t_0(\varphi_2) - t_0(\varphi_1) = \frac{Q_{t0}(\varphi_2) - Q_{t0}(\varphi_1)}{C_{\varphi 1}} . \tag{2}$$

The near-surface temperature of the Earth $C_{\varphi 1}$, average along the latitude φ and all meridians, is determined by the aggregate of physical, chemical, and geographic processes occurring on the Earth surface at the latitude φ. Therefore, we can consider that, in the case of invariance in the properties of this surface, the heat capacity $C_{\varphi 1}$ will be the same in a different epoch. Then, the near-surface temperature $t(\varphi)$ in the epoch T can be determined by the current temperature $t_0(\varphi)$ and the change in the quantity of heat $Q_T(\varphi) - Q_{T0}(\varphi)$:

$$t(\varphi) = t_0(\varphi) + \frac{Q_T(\varphi) - Q_{T0}(\varphi)}{C_{\varphi 1}} , \tag{3}$$

where $t_0(\varphi)$ is the temperature distribution along the latitude φ in the modern epoch, and $Q_T(\varphi)$ is the quantity of solar heat supplied to the Earth in the epoch T. The accuracy of expression (3) is determined by the level of accuracy of determining the heat capacity $C_{\varphi 1}$ at this latitude φ. According to (2), in the modern epoch, it can be calculated using the differences in heat $Q_{t0}(\varphi_2) - Q_{t0}(\varphi_1)$ and the temperature $t_0(\varphi)$ at these latitudes:

$$C_{\varphi 1} = \frac{Q_{t0}(\varphi_2) - Q_{t0}(\varphi_1)}{t_0(\varphi_2) - t_0(\varphi_1)} , \tag{4}$$

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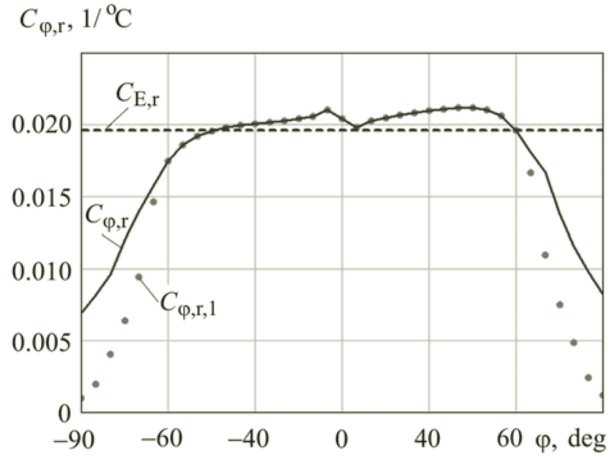

Fig. 1. Distribution of the relative near-surface heat capacity of the Earth along its latitude at $C_{E,r} = 1.9633\cdot10^{-2}$ 1/°C: the solid line is $C_{\varphi,r}$; the dots indicate $C_{\varphi,r,1}$.

where the surface heat capacity is determined at the intermediate latitude $\varphi = 0.5(\varphi_1 + \varphi_2)$.

For the annual insolation $Q_{t0,i}$ and the generalized temperature t_{oi}, in the modern epoch T, we have subscripted variables which are represented with an interval of 5° of the latitude in $I_f = 37$ points starting with 90°. To calculate the heat capacity $C_{\varphi 1i}$, the annual insolation $Q_{t0,i}$ was determined in $I_{f3} + 1 = 73$ points with an interval of 2.5°: $Q_{t0,i2}$, $i_2 = 1, 2, \ldots, I_{f3} + 1$. These $Q_{t0,i2}$ values are used to calculate the differences $\Delta Q_{t0}(\varphi_i)$ with an interval of 5° of the latitude.

As has already been pointed out, the temperature t_{oi} profile is a discrete function determined as a result of a number of approximations. Therefore, the temperature differences Δt_{oi} at the neighboring latitudes φ_1 and φ_2 are not smooth (continuously differentiable) functions. Therefore, it is necessary to smooth heat capacity to prevent its jumps at each latitude. It has been established that this smoothed heat capacity does not differ, in practice, from the heat capacity in which the temperature differences Δt_{oi} are obtained on the approximation temperature $t_{o,a}$:

$$\Delta t_o(\varphi) = (dt_{o,a}/d\varphi)\Delta\varphi , \tag{5}$$

where the derivative $dt_{o,a}/d\varphi$, according to (2) in [1], is determined as

$$dt_{o,a}/d\varphi = k_2 k_3 \cos(k_3\varphi + k_4) + k_5 k_6 \cos(k_6\varphi + k_7) . \tag{6}$$

As has already been noted, the approximation profile of the temperature $t_{o,a}$ does not account for the bulge in the profile t_{oi} in the equatorial latitudes of the Northern hemisphere (Fig. 1 in [1]). This bulge will be taken into account in the profile t_{oi} which is used in expression (3) for determining the temperate profile of a different epoch. Therefore, the use of Δt_o, according to (5), does not result in appreciable errors of temperature determination. Then, according to (4), the heat capacity in terms of the subscripted variable and the nondimensional annual insolation $Q_{t,nd}$, will be written as

$$C_{\varphi,r,1i} = \Delta Q_{t,nd,0}(\varphi_i)/\Delta t_o(\varphi_i) , \tag{7}$$

where the relative surface heat capacity

$$C_{\varphi,r,1i} = C_{\varphi,1i}/Q_{t0}^{45} . \tag{8}$$

The difference in the insolations $Q_{t,nd,0}(\varphi_i) = Q_{t,nd,0}(\varphi_{i-1}) - Q_{t,nd,0}(\varphi_{i+1})$ is determined at the latitude φ_i by the neighboring latitudes. As seen from Fig. 2a in [1], at the equator and poles, the insolation $Q_{t,nd,0}$ develops horizontal portions, therefore, $\Delta Q_{t,nd,0} \approx 0$. Due to this, the heat capacity in these portions is determined as a half-sum of heat capacities in appropriate points:

$$C_{\varphi,r,1,1} = 0.5(C'_{\varphi,r,1,1} + C'_{\varphi,r,1,2}) , \quad C_{\varphi,r,1,18} = 0.5(C'_{\varphi,r,1,17} + C'_{\varphi,r,1,19}) ,$$
$$C_{\varphi,r,1,37} = 0.5(C'_{\varphi,r,1,36} + C'_{\varphi,r,1,37}) , \tag{9}$$

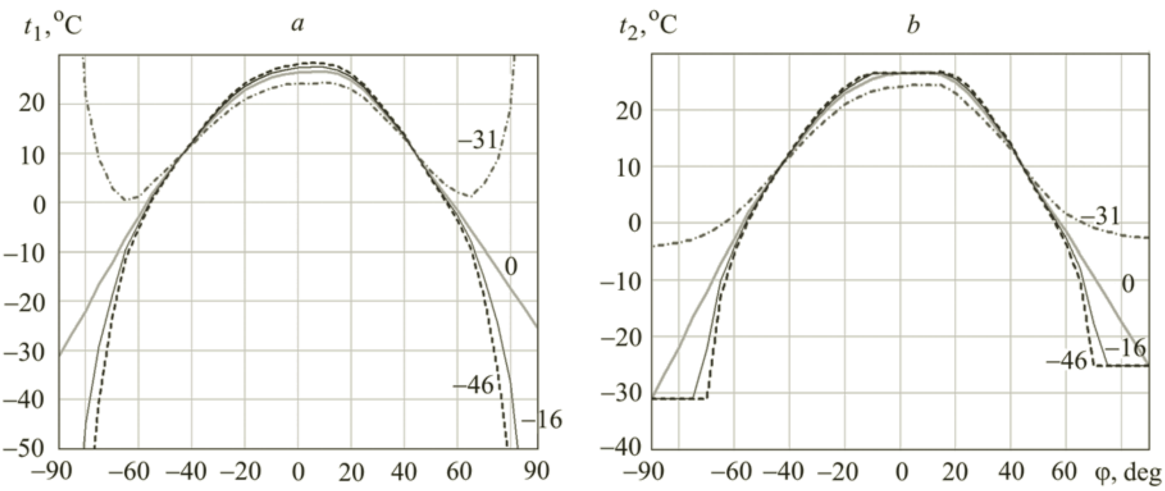

Fig. 2. Profiles of the annual temperatures of the Earth t_1 (a) and t_2 (b) in the modern epoch $T = 0$ kyr and in the epochs $T = –15.88$, –21.28, and –46.44 kyr.

where primed heat capacities are determined by (4). In Fig. 1, dots are used to show the distribution of the surface heat capacity $C_{\varphi,\mathrm{r},1}$ along a latitude of the Earth. Its maximum change occurs at high altitudes $|\varphi| > 60^{\mathrm{o}}$, and at the altitudes $|\varphi| < 60^{\mathrm{o}}$, its values are near 0.021/°C.

According to (8), temperature profile (3) in any epoch will be written in terms of the relative heat capacity $C_{\varphi,\mathrm{r},1i}$ as

$$t_1(\varphi_i) = t_{\mathrm{o},i} + \frac{Q_{\mathrm{t,n,d}}(\varphi_i) - Q_{\mathrm{t,n,d},0}(\varphi_i)}{C_{\varphi,\mathrm{r},1i}} . \tag{10}$$

Formula (10) was used to calculate the average annual temperature profiles $t_1(\varphi_i)$ in the modern epoch and in the three extreme epochs $T = 15.88$, 31.28, and 46.44 thousand years ago (Fig. 2). As follows from (10), the temperature profile $t_1(\varphi_i)$ in the modern epoch coincides with the profile $t_{\mathrm{o}i}$. It is seen from Fig. 2, that at low latitudes ($|\varphi| \le 45^{\mathrm{o}}$), temperature profiles differ insignificantly from each other. At high latitudes, they differ substantially. For example, at the North Pole, the temperature varies from 127°C in the epoch $T = 31.28$ thousand years ago to –190°C in the epoch $T = 46.44$ thousand years ago. Such significant changes in the near-surface temperature are indicative of the need for adjustment of the heat capacity for high latitudes determined by (7).

The annual insolation profiles in the equivalent latitudes $I_{\mathrm{t}}(\varphi)$ shown in Fig. 2b in [1] make it possible to determine temperature profiles of epochs by the current temperature profile $t_{\mathrm{o}}(\varphi)$. For this purpose, it is necessary to substitute $I_{\mathrm{t}}(\varphi)$ into the temperature profile $t_{\mathrm{o}}(\varphi)$ instead of the latitude φ:

$$t_2(\varphi) = t_{\mathrm{o}}(I_{\mathrm{t}}(\varphi)) . \tag{11}$$

However, it is impossible to perform such a simple operation for subscripted variables of the temperature $t_{\mathrm{o}i}$ and insolation $I_{\mathrm{t}i}$, since the insolation values in the equivalent latitudes $I_{\mathrm{t}}(\varphi)$ can be different and fail to coincide with the discrete φ_i values for which the temperature $t_{\mathrm{o}i}$ is assigned. Therefore, for each latitude φ_i in the epoch T, it is necessary to find index I_{n_i} which will correspond to the current latitude $\varphi_{I_n,i}$ with a close value to the insolation $I_{\mathrm{t},I_{n_i}}$. Then, this value and the neighboring one can be used to find a temperature corresponding to the insolation at the latitude φ_i by linear interpolation. With account for this condition, the temperature for the Northern hemisphere will be determined as

$$t_{2,\mathrm{N}}(\varphi_i) = t_{\mathrm{o}I_{n_i-1}} - \frac{t_{\mathrm{o}I_{n_i-1}} - t_{\mathrm{o}I_{n_i}}}{\varphi_{I_{n_i-1}} - \varphi_{I_{n_i}}} (\varphi_{I_{n_i-1}} - I_{\mathrm{t},i}) , \tag{12}$$

where $\varphi_i = 90, 85, \ldots, 5, 0^{\mathrm{o}}$. The value of the index I_{n_i} for the latitude φ_i is calculated for the insolation $I_{\mathrm{t}i}$ to be found between the neighboring latitudes in the points $I_{n_{i-1}}$ and I_{n_i}. This makes it possible to use the $I_{\mathrm{t}i}$ value for interpolating temperature by the temperature in the points $I_{n_{i-1}}$ and I_{n_i}.

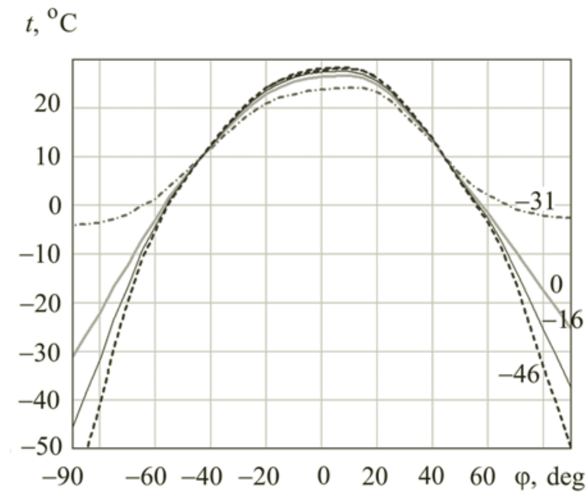

Fig. 3. Distribution of the average annual near-surface temperature of the Earth t calculated by the heat capacity $C_{\varphi,r}$ and the annual insolation $Q_{t,nd}$, along its latitude in the modern epoch $T = 0$ kyr and in the epochs $T = -15.88$, -21.28, and -46.44 kyr.

According to Fig. 1 in [1], the profile of the temperature t_{oi} has the maximum in the Northern hemisphere at $\varphi = 10^o$. Expression (12) is written for the rising branch t_{oi}, and for the descending arm, including the Southern hemisphere; the temperature is determined by the formula

$$t_{2,S}(\varphi_{Si}) = t_{o,I_{Sn\,i}} - \frac{t_{o,I_{Sn_i}} - t_{o,I_{Sn_{i+1}}}}{|\varphi_{I_{Sn_i}}| - |\varphi_{I_{Sn_{i+1}}}|}(|\varphi_{I_{Sn_i}}| - I_{t,S,i}) \,, \tag{13}$$

where $\varphi_{Si} = 10, 5, 0, -5, -10, \ldots, -90^o$. Here, the index I_{n_i} is also calculated in such a way that the insolation $I_{t,I_{n_i}}$ is found between neighboring latitudes. According to formulas (12) and (13), at the overlapping segments $\varphi = 10$ and 5^o, the temperatures differ. Therefore, they are calculated by the formulas

$$t_2 = 0.3t_{2,N} + 0.7t_{2,S} \,. \tag{14}$$

The coefficients 0.3 and 0.7 in formula (14) are determined from the condition of equality of the temperature t_2 and temperature t_o in the modern epoch.

The profiles of the temperature t_2 determined by (12)–(14) for four epochs are shown in Fig. 3. As seen from Fig. 2b in [1], in cold epochs, insolation in the equatorial region is limited $I_t = 0^o$, and at high latitudes $I_t = 90^o$. Therefore, the temperatures t_2 in Fig. 3 are also limited at these latitudes: 26.46°C in the equatorial zone, –25°C in the North Pole zone, and –31°C in the South Pole area. In the cold epochs $T = 15.89$ and 46.44 thousand years ago, temperature profiles have these limitations in the equatorial and polar latitudes. And in the warm epoch $T = 31.28$ thousand years ago, the temperature t_2 does not have such limitations. Therefore, this variant represents temperature without distortion. The temperature profiles $t_2(\varphi)$ (Fig. 3) in all epochs coincide with the profiles $t_1(\varphi)$ (Fig. 2) in the region of latitudes $|\varphi| \le 60^o$, excluding the equatorial region for the cold epochs $T = 15.88$ and 46.44 thousand years ago.

As has already been pointed out, there is a need for correction of the heat capacity $C_{\varphi,r,1}$ in the high-latitude region. It can be performed using an undistorted temperature profile t_2 in the warm epoch $T = 31.28$ thousand years ago. Then, by analogy with (4) and (7) we write the expression

$$C_{\varphi,r,2,i} = \frac{Q_{t,nd,31}(\varphi_i) - Q_{t0}(\varphi_i)}{t_{2,31}(\varphi_i) - t_0(\varphi_i)} \,, \tag{15}$$

where $Q_{t,nd,31}(\varphi_i)$ and $t_{2,31}(\varphi_i)$ are the nondimensional annual insolation and temperature in the epoch 31.28 thousand years ago.

To smooth the alignment of the profiles of the heat capacities $C_{\varphi,r,1}$ and $C_{\varphi,r,2}$ in the transition points, the average heat capacity is calculated in these points:

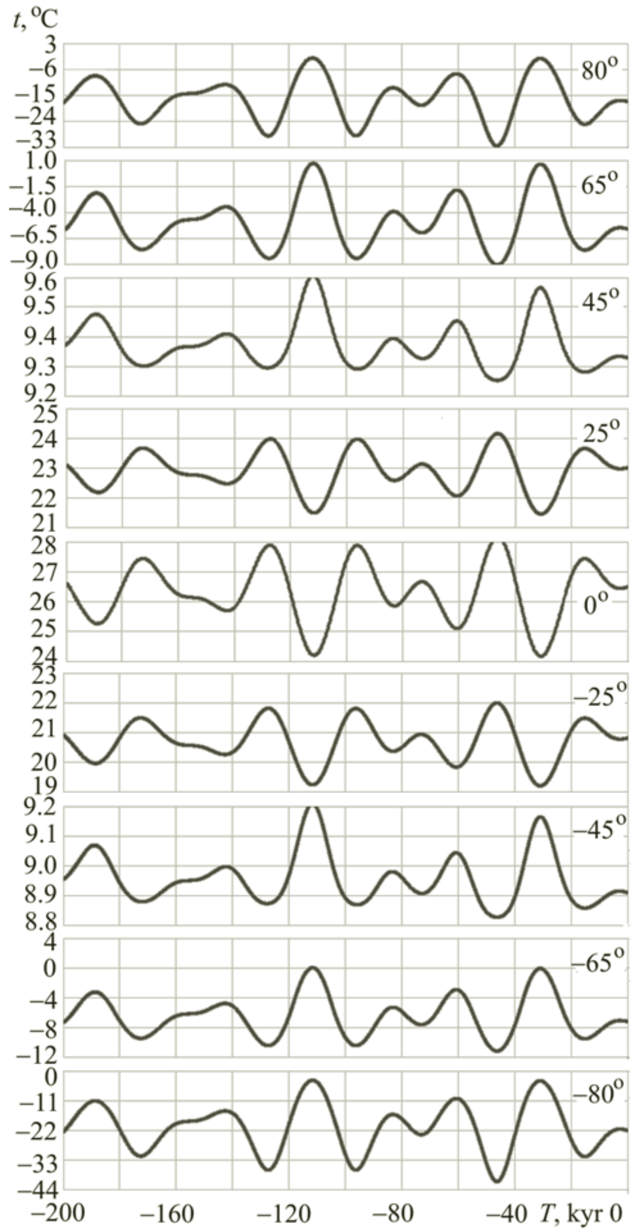

Fig. 4. Evolution of the average near-surface temperature of the Earth t in the past 200 thousand years at different latitudes of the Earth.

$$C_{\varphi,\mathrm{r},i} = 0.5(C_{\varphi,\mathrm{r},1,i} + C_{\varphi,\mathrm{r},2,i}) \text{ at } |\varphi_i| = 65^{\circ} . \tag{16}$$

Thus, the corrected near-surface heat capacity of the Earth $C_{\varphi,\mathrm{r},i}$ is determined by expressions (7), (15), and (16). The profile $C_{\varphi,\mathrm{r}} = C_{\varphi,\mathrm{r},i}$ is represented by a solid line in Fig. 1 and is shown in Table 2 of investigation [1]. The Earth surface-average near-surface heat capacity calculated in accordance with (5) of investigation [1] is equal to $C_{\mathrm{E,r}} = 1.9633 \cdot 10^{-2}$ 1/°C. The near-surface heat capacity in the dimensional form $C_{\varphi,i}$ is determined by the multiplication of the relative heat capacity $C_{\varphi,\mathrm{r},i}$ by the annual insolation Q_{t0}^{45} at the latitude $\varphi = 45^{\circ}$, i.e.,

$$C_{\varphi,i} = C_{\varphi,\mathrm{r},i} Q_{\mathrm{t0}}^{45} . \tag{17}$$

With account for (17), the Earth surface-average heat capacity in the dimensional form will be determined as

TABLE 1. Near-Surface Average Annual Temperatures at the Equator and Poles of the Earth in Four Epochs

T, kyr	t, °C		
	φ, deg		
	0	90	–90
0	26.5	–25.3	–31.1
–15.88	27.4	–37.4	–45.6
–31.28	24.2	–2.65	–4.09
–46.44	28.2	–49.7	–60.2

TABLE 2. Average Near-Surface Paleotemperatures of the Earth and Its Hemispheres in Four Epochs

T, kyr	t_{NH}, °C	t_{SH}, °C	t_E, °C
0	15.10	13.73	14.41
–15.88	14.93	13.46	14.19
–31.28	15.44	14.32	14.88
–46.44	14.77	13.21	13.99

$$C_E = C_{E,r} Q_{t0}^{45} = 1.95 \cdot 10^5 \text{ kJ/(m·°C)} . \quad (18)$$

In the case of the corrected heat capacity $C_{\varphi,r,i}$, the temperature profiles $t(\varphi)$ in any epoch T are determined by expression (10), where $C_{\varphi,r,i}$ are substituted for $C_{\varphi,r,1,i}$. In Fig. 3, these temperature profiles are presented for four epochs. In the equatorial region, the annual temperatures differ by 4°C. At the latitude of 45°, the temperatures do not change. After φ = 60°, the profile difference increases and, at the poles, they differ by 50–60°C. Table 1 shows the values of temperatures at the equator and the poles for these four epochs.

The average annual temperatures shown in Fig. 3 were used to calculate the average temperature of the Earth and hemispheres for the four epochs (Table 2) with the help of expressions (5) and (6) in [1]. In the warmest epoch T = 31.28 thousand years ago, the Earth and hemisphere temperatures are approximately 0.4°C higher compared to the modern epoch, and in the cold epoch T = 46.44 thousand years ago, they are as much lower. With the virtually unchanged insolations, in the considered epochs, these minor temperature changes can exist due to the variation in the latitude temperature distribution shown in Fig. 3. Therefore, the data in Table 2 are an indirect confirmation of the adequacy of midlatitude temperatures in Fig. 3.

The average paleotemperature of the Earth in the epoch T can be estimated by the near-surface heat capacity of the Earth $C_{E,r}$:

$$t_E(T) = t_{E,0} + \frac{Q_{t,nd,E}(T) - Q_{t,nd,E,0}}{C_{E,r}} , \quad (19)$$

where $t_{E,0}$ and $Q_{t,nd,E,0}$ are the Earth temperature and the nondimensional annual insolation respectively in the modern epoch. According to Table 1 in [1], $Q_{t,nd,E}(T)$ of the said four epochs does not differ, in practice, from $Q_{t,nd,E,0}$. Therefore, in accordance with (19), the Earth temperature in these epochs does not change. An estimate by formula (19) with an accuracy of ±0.45°C coincides with calculations of the average paleotemperature of the Earth by the midlatitude temperature shown in Table 2. Therefore, in cases when the insolation $Q_{t,nd,E}(T)$ differs from the current insolation $Q_{t,nd,E,0}$, we can use formula (19) to calculate the Earth temperature with an expected error of ±0.45°C.

Temperature Evolution on the Earth Surface. Investigations [2–4] cite changes in the quantity of annual heat Q_t during 200 thousand years, which makes it possible to determine the Earth temperature evolution in this time interval by formula (10). Furthermore, in (10), instead of the original heat capacity $C_{\varphi,r,1,i}$, use is made of the corrected heat capacity $C_{\varphi,r,i}$. Formula (10) includes the subscripted variables t_{oi}, $Q_t(\varphi_i)$, and $C_{\varphi,r,i}$ in points of the latitude φ_i. To determine the temperature at any latitude in the internal between the φ_i points, we make use of a linear interpolation of these values. For example, the heat capacity at the latitude in the interval from φ_i to φ_{i+1} is determined as

$$C_{\varphi,r} = C_{\varphi,r,i} + \frac{C_{\varphi,r,i+1} - C_{\varphi,r,i}}{\varphi_{i+1} - \varphi_i} (\varphi - \varphi_i) . \tag{20}$$

Similarly to (20), the temperature $t_o(\varphi)$ and the nondimensional annual insolation in the modern epoch $Q_{t,nd,0}(\varphi)$ are determined at any latitude. Then, instead of (10), the temperature in any epoch T at any latitude will be determined by the formula

$$t(T, \varphi) = t_o(\varphi) + \frac{Q_{t,nd}(T, \varphi) - Q_{t,nd,0}(\varphi_i)}{C_{\varphi,r}} , \tag{21}$$

where $Q_{t,nd}(T, \varphi)$ is the nondimensional annual insolation in the epoch T at the latitude φ.

Figure 4 shows changes in the average annual near-surface temperature of the Earth during 200 thousand years at its nine latitudes. The latitude of 65 N (65 degrees of latitude north of the equator) is taken by M. Milankovich as an indicator of climate change [5]. In the modern epoch (T = 0), the average annual temperature is equal to –5.6°C at this latitude. Then it increases and there is a slight optimum in it in the epoch T = 2.8 thousand years ago, after which the temperature decreases down to –7.62°C in the epoch T = 15.32 thousand years ago. This is the middle of the latest ice period. After it, the temperature rises to 0.66°C in the epoch T = 31 thousand years ago. This is the middle of a warm period which is called Karginsky Interglacial in Western Siberia [3]. After the warm period, the temperature drops, and in the epoch T = 46.4 thousand years ago decreases to the temperature minimum of –9.05°C of the penultimate ice period. This is the coldest period in the 200 thousand years. And the warmest epoch was the epoch T = 111.8 thousand years ago with the temperature of 0.75°C. Thus, at the above-indicated latitude, the temperature varies from 0.75 to –9.05°C, i.e., the fluctuations range is 9.8°C.

It should be noted that the optima established by the summer insolation Q_s at 65 N [3], occur in epochs with a slight difference, for example, in T = 4.16, 15.88, 31.28, and 46.44 thousand years ago, which corresponds to the Holocene optimum and the middles of the latest glaciation (glacial period), the warm period, and the penultimate ice period respectively. At the latitude of 80°, the time of occurrence of the optima of these events also changes slightly. At 80 N, the said temperature fluctuations occur again but with a higher amplitude: from –33.1°C in the epoch T = 46.4 thousand years ago to –2°C in the warm epoch T = 111.8 thousand years ago, i.e., the fluctuations range here is 31°C, which is three times higher than at the latitude of 65°C. At 45 N, temperature fluctuations are similar to temperature fluctuations at the latitudes of 65 and 80°, but the fluctuations range is significantly smaller: from 9.25°C in the epoch T = 46.4 thousand years ago to 9.61°C in the epoch T = 111.8 thousand years ago, i.e., the range is 0.36°C. This is 27 times less compared to the temperature fluctuations range at the latitude of 65°. Thus, at the latitude of 45°, the temperature does not change, in practice. At the latitude of 25°, temperature fluctuations that are phase-inverse to fluctuations at the latitudes 45–80°, i.e., in cold epochs it becomes colder at high latitudes, and at low latitudes it becomes warmer. This trend continues to be observed at the equator (φ = 0°) and at –25 S (25 degrees of latitude south of the equator). In this case, the temperature fluctuations range does not exceed 4°C. It is almost three times less than the temperature fluctuations range at the latitude of 65°. At the latitudes from –45 to –80° south of the equator, the temperature fluctuations pattern is identical to that at the same latitudes of the Northern hemisphere, i.e., coolings and warmings at the said latitudes occur simultaneously. There are small differences in temperature values that increase with increase in latitude. For example, at the latitude of 80°, the temperature is equal to –3.38°C in the warm epoch T = 111.8 thousand years ago, and in the cold epoch T = 46.4 thousand years ago, the temperature is –40.94°C. In this epoch, at the same latitude of the Northern hemisphere, this minimal temperature is equal to –33.1°C, i.e., it is colder in the Southern hemisphere.

Discussion of Results. *Comparison of the obtained data with the findings of other authors and observational results.* Figure 5 shows the profile of the annual near-surface temperature of the Northern hemisphere of the Earth constructed on the basis of the data from 927 meteorological stations using the average monthly near-surface temperature of the Earth for 1955–2014 [6]. The midlatitude temperature obtained in [6] was calculated by the relation

$$t_s(\varphi) = A_s s_n(\varphi) - B_s , \tag{22}$$

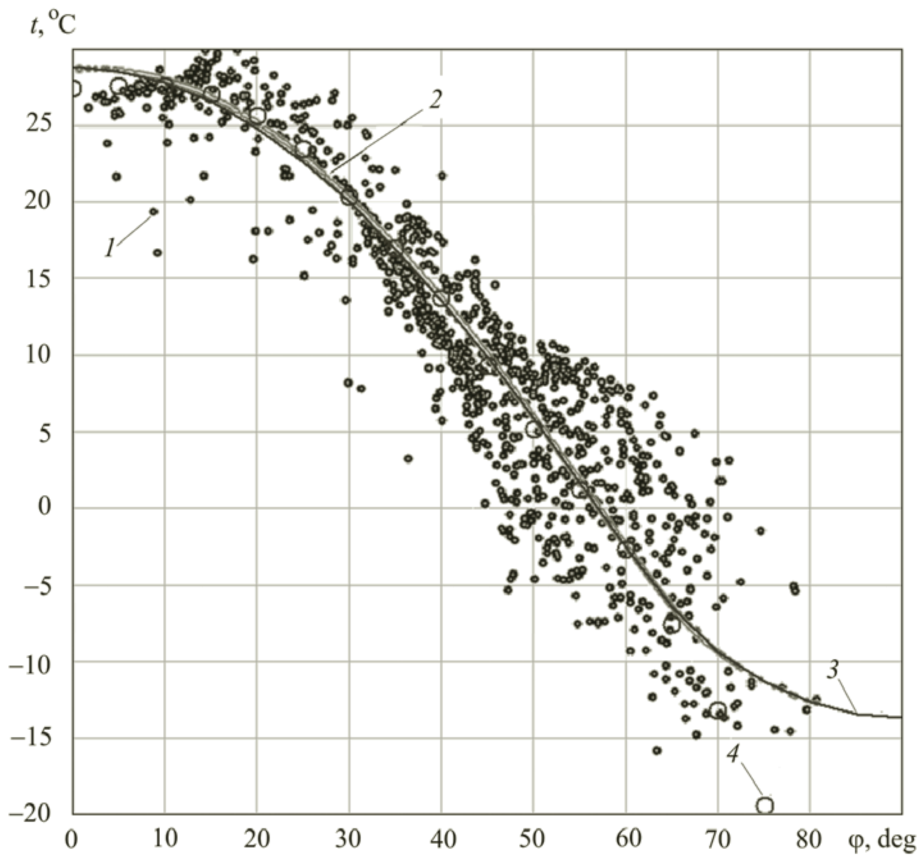

Fig. 5. Comparison of different distributions of the average annual near-surface temperature of the Earth t along its latitude: 1) observations at 927 meteorological stations of the Northern hemisphere for 1955–2014 [6]; 2) linear correlation of the dependence $t_s(\varphi)$ [7]; 3) linear correlation of the dependence $t_Q(\varphi)$ with regard to the nondimensional insolation $Q_{\mathrm{t,nd,0}}(\varphi)$ according to (23); 4) generalized temperature profile t_{oi}.

where $s_n(\varphi)$ is the nondimensional annual insolation obtained by relating the annual insolation to its value in the perihelion point, $A_s = 225.030^{\mathrm{o}}$C and $B_s = 39.005^{\mathrm{o}}$C. The coefficients A_s and B_s are obtained as a result of minimization of the total squared error $t_s(\varphi)$ in the temperatures calculated by the results of observations. It should be noted that in investigations of the authors [6], calculations were performed for several variants. In Fig. 5, line 2 shows the variant from the Report by these authors [7]. The correlation dependence (22) in the variables used in this investigation has the form

$$t_Q(\varphi) = A_Q Q_{\mathrm{t,n,d,0}}(\varphi) - B_Q \ , \tag{23}$$

where $A_Q = 53.5690^{\mathrm{o}}$C and $B_Q = 43.7293^{\mathrm{o}}$C. The temperature profile $t_Q(\varphi)$ calculated by (23) (line 3) almost coincides with correlation (22) (line 2). Line 2 is shown with a dashed line outside the latitude range from 10 to 70$^{\mathrm{o}}$, i.e., dependence (22) does not reflect the temperature change in the equatorial and polar regions. In the latitude range from 10 to 65$^{\mathrm{o}}$, the temperature t_{oi} coincides, in practice, with temperature profiles 2 and 3. Therefore, in this latitude region, the midlatitude temperature can be determined by formula (23).

Local average annual paleotemperature. As seen from Fig. 5, in the modern epoch, the local average annual temperatures $t_{\mathrm{loc,0}}(\varphi)$ can differ substantially at one and the same latitude. For example, at $\varphi = 57^{\mathrm{o}}$, the local average annual temperatures vary from 7 to –7$^{\mathrm{o}}$C at $t_{\mathrm{o}} = 0.4^{\mathrm{o}}$C. For the modern epoch, we introduce the difference between local temperature and average temperature

$$\Delta t_{\mathrm{loc,0}}(\varphi) = t_{\mathrm{loc,0}}(\varphi) - t_{\mathrm{o}}(\varphi) \ . \tag{24}$$

TABLE 3. Midlatitude $t(\varphi)$ and Local $t_{loc}(\varphi)$ Annual Paleotemperatures at the Latitude of the City of Tyumen and the City of Moscow in Four Epochs

T, kyr	Tyumen, $\varphi = 57.152199^o$		Moscow, $\varphi = 55.75222^o$	
	t, ^oC	t_{loc}, ^oC	t, ^oC	t_{loc}, ^oC
0	0.42	1.5	1.31	5
–15.88	–0.43	1.08	0.59	4.28
–31.28	2.97	4.05	3.45	7.14
–46.44	–1.07	0.01	0.05	3.74

In the assumption that in a different epoch the geophysical characteristics of the terrain will not change substantially, we estimate the local average annual temperature with account for this difference:

$$t_{loc}(\varphi) = t(\varphi) + \Delta t_{loc,0}(\varphi) . \quad (25)$$

The profile t_{oi} was used for interpolating the midlatitude temperatures $t(\varphi)$ of the cities of Tyumen and Moscow for four epochs (Table 3). According to the Hydrometeorological Center of Russia, the average annual temperatures at meteorological stations of Tyumen and Moscow (VDNKh, Exhibition of Achievements of National Economy) was 1.5 and 5^oC respectively in 1961–1990. In Table 3, they are shown as local temperatures in the epoch $T = 0$. The temperatures shown in Table 3 are higher than the midlatitude temperatures $t(\varphi)$ by 1.1^oC for Tyumen and by 3.7^oC for Moscow. It appears that in these examples, this difference is largely due to urban heat emissions rather than to the geophysical characteristics of the terrain. Therefore, to estimate local paleotemperatures, it is necessary to use current local temperature from meteorological stations where there are no technogenic sources of temperature change. Nevertheless, at the extreme values of the ice epochs $T = 15.88$ and $T = 46.44$ thousand years ago, the midlatitude temperature $t(\varphi)$ is negative at the latitude of Tyumen, i.e., permafrost rocks can develop here. And the local temperatures $t_{loc}(\varphi)$ are positive, therefore there will be no development of permafrost in this terrain. Thus, an estimate of local temperatures makes it possible to identify the diversity of paleoclimate in different localities situated at one and the same latitude.

Example of using the results of determination of the near-surface heat capacity of the Earth. Since the near-surface heat capacity of the Earth was determined by two methods using objective thermophysical characteristics of the Earth, it can be used for estimating thermophysical phenomena. Tracer (labeled-atom) investigations have shown that three billion years ago, the Earth temperature could reach 70^oC [8]. Using near-surface heat capacity of the Earth $C_{E,r}$, we estimate the quantity of heat supplied to the Earth required for achieving the said temperature. According to (19), the annual insolation in the epoch T can be determined by the formula

$$Q_{t,nd,E}(T) = Q_{t,nd,E,0} + (t_E - t_{E,0})C_{E,r} . \quad (26)$$

After substituting $Q_{t,nd,E,0} = 1.1087$, $t_E = 70^oC$, $t_{E,0} = 14.41^oC$, and $C_{E,r} = 1.9633 \cdot 10^{-2}$ $1/^oC$ into (26), we obtain the nondimensional annual insolation of the Earth $Q_{t,nd,E}(T) = 2.2$, which is equal to $1.984Q_{t,nd,E,0}$, i.e., 3 billion years ago, the Earth received almost twice as much solar heat as now.

As follows from (12) in [1], the annual quantity of heat is determined by three parameters, viz., J_0, $P_{tr,sc}$, and $R_{E,m}$. In the case of the unchanged period of the Earth's revolution $P_{tr,sc}$ and its radius $R_{E,m}$, the annual insolation of the Earth depends only on the solar constant J_0. According to (12) in [1], the solar constant $J_0(T)$ in the epoch T is equal to

$$J_0(T) = \frac{Q_{t,nd,E}(T)Q_{t0}^{45}}{\pi R_{E,m}^2 P_{tr,sc}} . \quad (27)$$

As is seen from (27), with a twofold increase in $Q_{t,nd,E}(T)$ the solar constant $J_0(T)$ rises two times too. Since the quantity of heat radiated by the sun is proportional to its brightness and area, the said rise can take place with a twofold increase in the solar brightness, with a 1.414fold increase in its diameter or in the case of simultaneous increase of the first and second parameters in a number of times corresponding to the area and brightness of the Sun.

Conclusions. The near-surface heat-capacity of the Earth distributed along its latitude is determined by two methods: by the latitude variation in the Earth temperature and by the Earth insolation in the modern epoch and its variation with time. Based on the obtained data, profiles of the midlatitude paleotemperature of the Earth in three extreme epochs during 50 thousand years have been constructed. Variation in the Earth paleotemperature during 200 thousand years at its nine latitudes has been calculated. The generalized midlatitude temperature of the Earth has been compared with new results of other investigators. An estimate of the local paleotemperature of the Earth has been given, and a possible use of surface heat capacity in other paleoclimate investigations has been shown.

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NOTATION

$C_{E,r}$, relative near-surface heat capacity of the Earth, its surface average; $C_{\varphi,1}$, near-surface heat capacity of the Earth calculated by the latitude variation of its temperature; t_1, annual temperature of the Earth calculated by the heat capacity $C_{\varphi,r,1}$; t_2, annual temperature of the Earth calculated by the annual insolation of the Earth in the equivalent latitudes I_t.

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