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Periodic Orbits of *N* Bodies on a Sphere

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Abstract—Gravitational interactions of *N* bodies are considered, which form a structure distributed over a sphere. A method and program for creating such structures are developed based on an exact solution to the problem of the axisymmetric interaction of *N* bodies. Studies on creating the structures are conducted, and their dynamics and evolution are investigated. On this basis, the dynamics and evolution of globular star clusters are explained.

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1. INTRODUCTION

In contemporary astrodynamics, the statistical approach to evolution of associations of stars is predominantly used. Based on several simplifications or hypotheses, a gravitational potential of association Φ is introduced [1, 2]. The probability of finding a star at a certain point in space and at a certain velocity is expressed by the function of phase space density ψ. Since potential Φ depends on the distribution of bodies in space, then from the dependence of Φ on ψ, different (collisional, collisionless, etc.) equations of astrodynamics are deduced. Then, a problem of statistical astrodynamics is the selection of the function of phase space density ψ (using the astrodynamics equations) such that it would yield potential Φ, which could provide the observed parameters of the association of stars.

It is accepted that potential Φ creates regular forces, while impacts on each star from individual stars create irregular forces. Therefore, the function of phase space density ψ, defined in the first problem, should be refined with allowance for action of irregular forces.

However, since several simplifications are introduced in this approach, the star association, obtained with distribution function ψ, should be verified by numerical integration of differential equations of the motion of all stars. As a rule, this verification shows that the desirable aim is not achieved. As a result, different explanations are advanced and additional factors of action are introduced. Searching for them and taking them into account generate new problems.

This study considers a deterministic approach rather than a statistical one as applied to globular star clusters. An association of stars is specified deterministically: each star has its own mass, radius, coordinates, and velocities. The evolution of the entire association is studied as a result of solving differential equations of motion of each star. With this approach, a certain idea is necessary: how to specify parameters (masses, coordinates, and velocities) of the association of stars for the calculated motions of bodies of this association to really represent the evolution of the observed association.

Thousands of stars, which are included in globular star clusters, are attracted to one another according to Newton's law of universal gravitation. Their existence raises two questions. If all stars are attracted to one another, then why they do not merge into one body? This can be explained by the fact that in the velocity of each star there is a component perpendicular to the total force; therefore, the star's motion occurs along a curvilinear trajectory. As a result, all stars are in quasiperiodic motion. Precisely in this way, the planets move around the Sun. Despite the fact that each planet is attracted by the Sun and other planets, it can forever revolve around the Sun in a quasielliptic orbit.

In the solar system, the planets move almost in the same plane, and the orbits of the planets do not intersect. However, in a globular cluster, the planes of the orbits of the stars are located in space, and there are so many of them that it would seem that collisions are inevitable. As a result of collisions, the stars will merge into a single star, while with close passes, they will be ejected from the cluster. Therefore, with time the globular star cluster should disappear. However, they do not vanish, and astronomers refer to them as the oldest objects in galaxies. From here the second question arises: why do globular star clusters exist for a long time?

To answer these questions, it is necessary to consider the interaction of stars in the globular star cluster and to study their movements. To specify coordinates and velocities of stars, it is possible to take advantage of the experience of creating structures with interacting bodies that move periodically on the plane. For

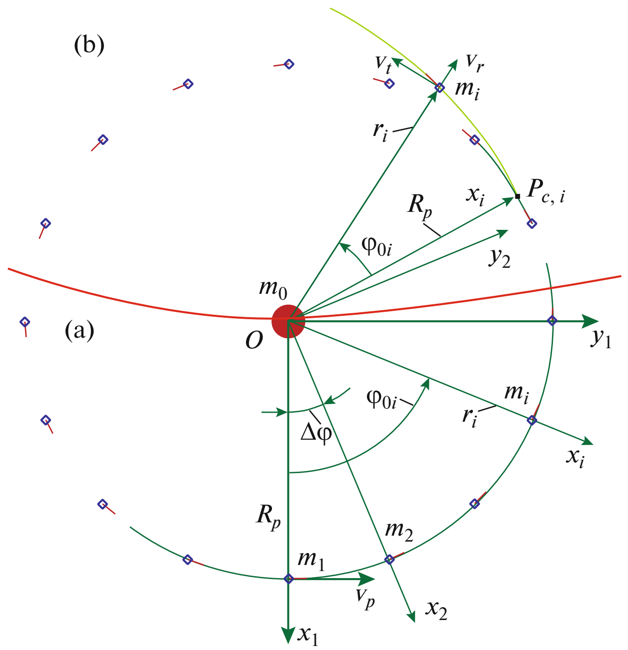

Fig. 1. Geometric characteristics of the planar single-layer axisymmetric structure of N bodies with central body mass m_0 and mass of each peripheral body $m_i = m_1$: (a) initial location of bodies in variants I, II, and III of spherically distributed structures ($\varphi_{0,i}$ is the polar angle of the body m_i from the x_1 axis); (b) initial location of bodies in variant IV ($\varphi_{0,i}$ is the polar angle of the body m_i measured from the pericenter of its orbit $P_{c,i}$).

these structures, exact solutions to two problems of gravitational interaction of N bodies have been obtained. In the first problem [3, 4], around a central body, $N - 1$ peripheral bodies are uniformly located along the circle. In this structure, depending on the velocity of peripheral bodies, they can move along an ellipse, parabola, and hyperbola. If there is only radial velocity, the structure, depending on the magnitude and sign of the velocity, turns into one body or the bodies in it, are removed to infinity.

In the second problem [5, 6], a multilayer rotating structure on a plane is considered, which consists of N_2 layers with N_3 bodies located on each of them. Due to the variation in radii of layers and in angles of mutual arrangement of the bodies in the neighboring layers, a countless number of varieties of these rotating structures can be created. In [3–6], the methods and programs of creating the structures are developed, while their dynamics and evolution were studied using the Galactica system [7–9].

In [5, 6] the possibility of turning flat rotating structures into spatial ones due to the rotation of layers in space is noted. In this study, a spatial structure is created due to using results of the first problem. For this purpose, in the single-layer axisymmetric structure, orbits of the bodies are deployed successively in space.

2. MAIN RESULTS OF THE PROBLEM OF AXISYMMETRIC INTERACTION OF N BODIES

In this problem [3, 4], N_3 bodies with a mass of $m_i = m_1$ are located axisymmetrically on a plane (see Fig. 1a) around the central body of mass m_0. Through body m_1, the x_1 axis passes. From it, polar angles $\varphi_{0,i}$ of the remaining bodies are counted:

$$\varphi_{0,i} = (i-1)\Delta\varphi, \quad i = 1, 2, \ldots, N_3, \tag{1}$$

where $\Delta\varphi = 2\pi/N_3$.

In polar coordinates r, φ_0, where r in Fig. 1a coincides with x_1, coordinates of body m_i will be r_i, φ_{0i}. As a result of solving the problem of Newton's interaction of bodies, the trajectory equation in the polar system of coordinates is derived in the following form [3, 4]:

$$r_i = \frac{R_p}{(\alpha_1 + 1)\cos\varphi_i - \alpha_1}, \tag{2}$$

where R_p is the pericenter radius, i.e., a point in orbit with the least distance to center O in Fig. 1;

$$\alpha_1 = \mu_1/\left(R_p v_p^2\right); \tag{3}$$

$$\mu_1 = -G\left(m_0 + m_1 f_{N_3}\right); \tag{4}$$

$$f_{N_3} = 0.25\sum_{i=2}^{N_3} \frac{1}{\sin\left[\pi(i-1)/N_3\right]}. \tag{5}$$

In Eqs. (2)–(5), φ_i is the polar angle of body m_i, counted from its pericenter; α_1 is the trajectory parameter; μ_1 is the interaction parameter; and f_{N_3} is the contribution of action of $N_3 - 1$ peripheral bodies on one of them. Depending on trajectory parameter α_1, the orbits of the peripheral bodies can be circles ($\alpha_1 = -1$), ellipses ($-1 < \alpha_1 < -0.5$), parabolas ($\alpha_1 = -0.5$), or hyperbolas ($-0.5 < \alpha_1 < 0$). The time of body motion along the trajectory also depends on α_1 [3, 4].

Let us write the other four orbit parameters of peripheral bodies [3]: orbital period

$$P = -\frac{2\pi\alpha_1 R_p}{v_p(-2\alpha_1 - 1)^{3/2}}, \tag{6}$$

pericenter velocity

$$v_p = \sqrt{(\mu_1/\alpha_1 R_p)}, \tag{7}$$

orbit eccentricity

$$e = -(1 + 1/\alpha_1), \tag{8}$$

and semimajor axis of orbit

$$a = R_p(2\alpha_1 + 1)/\alpha_1. \tag{9}$$

3. GEOMETRIC CONSTRUCTION OF A STRUCTURE OF N BODIES WITH PERIODIC ORBITS ON A SPHERE

Beginning from the second body (Fig. 1a), the plane of its orbit is turned around polar radius of the body $r_2 = x_2$ by angle $\Delta\theta$ together with all remaining bodies with radii r_3, r_4, ..., r_{N_3}. Then, orbits of bodies with r_3, r_4, ..., r_{N_3} are turned around radius r_3 of the third body by the same angle. If these turns are performed for all remaining peripheral bodies, then we obtain a structure of N bodies with periodic orbits on a sphere. Further on, we will call it a spherically distributed structure.

At the moment of construction, the bodies of an axisymmetric structure (Fig. 1a) are at pericenters with radius R_p and they have only transversal velocities v_p. Their radial velocities $v_r = 0$. Figure 2 shows the initial stage of structure construction. With body m_1, coordinate system $x_1y_1z_1$ is connected. Body m_2 in the x_1Oy_1 plane is shifted from body m_1 by angle

$$\Delta\psi = k_\varphi \Delta\varphi. \tag{10}$$

Coefficient k_φ is introduced for the possibility of varying the geometry of a spatial structure. We write coordinates and velocities of first body m_1 in coordinate system $x_1y_1z_1$:

$$x_{1,1} = R_P;\quad y_{1,1} = 0;\quad z_{1,1} = 0;$$
$$v_{x,1,1} = 0;\quad v_{y,1,1} = v_p;\quad v_{z,1,1} = 0. \tag{11}$$

In expressions (11) the first index "1" denotes the number of the coordinate system, while the second index "1" denotes the body number. With second body m_2, we connect coordinate system $x_2y_2z_2$ (Fig. 2). It is turned around the x_2 axis by angle

$$\Delta\theta = k_{\varphi v} \Delta\varphi. \tag{12}$$

Coefficient $k_{\varphi v}$ is also introduced for varying the spatial structure geometry.

Using angles $\Delta\psi$ and $\Delta\theta$, expressions for coordinates $x_{1,2}$, $y_{1,2}$, $z_{1,2}$ and velocities $v_{x,1,2}$, $v_{y,1,2}$, and $v_{z,1,2}$ of the second body are written in coordinate system $x_1y_1z_1$ of the first body. For the third body these transformations are executed in coordinate system $x_2y_2z_2$ of the second body. As a result of this successive consideration, transformations for the lth body from the coordinate system of the ith body to the coordinate system of the $(i-1)$th body are obtained [10]:

$$x_{i-1,l} = x_{i,l}\cos\Delta\psi - y_{i,l}\sin\Delta\psi\cos\Delta\theta + z_{i,l}\sin\Delta\psi\sin\Delta\theta; \tag{13}$$

$$y_{i-1,l} = x_{i,l}\sin\Delta\psi + y_{i,l}\cos\Delta\psi\cos\Delta\theta - z_{i,l}\cos\Delta\psi\sin\Delta\theta; \tag{14}$$

$$z_{i-1,l} = y_{i,l}\sin\theta + z_{i,l}\cos\Delta\theta. \tag{15}$$

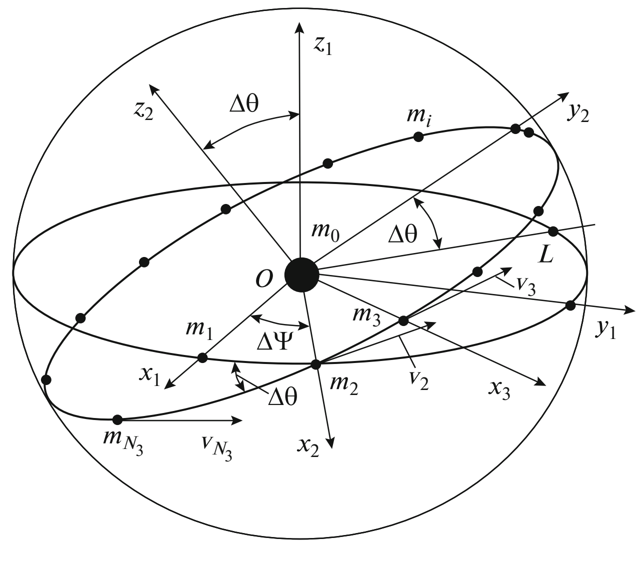

Fig. 2. Initial stage of construction of the spherically distributed structure: orbital planes of bodies from m_2 to m_{N_3} are turned by angle $\Delta\theta$ about the Ox_2 axis that passes through body m_2.

Similar expressions describe transformations for velocities. In Eqs. (13)–(15), for each index l, beginning from 2 to $N_3 - 1$, the coordinates of bodies from 3 to N_3 are presented in the coordinate system with a number that is one less. Therefore, by Eqs. (13)–(15), for a body with N_3, $(N_3 - 2)$ transformations should be performed, for a body with $(N_3 - 1)$, $(N_3 - 3)$ transformations should be executed, and so forth, up to the body of $l = 3$ with a single transformation. Then, the 3D-rotated coordinates and velocities of all bodies will be expressed in coordinate system $x_1y_1z_1$.

4. STRUCTURE CONSTRUCTION PROGRAM

This algorithm of structure construction (at first sight, very simple) includes some nontrivial problems. It is implemented in the SphDsSt4.for program [10]. The program consists of three parts: (1) reading initial parameters; (2) constructing a spherically distributed structure; and (3) creating the input file for the Galactica system.

The main initial parameters are read from the SphDsSt4.dat data file [10]. In this file, the structure parameters are specified: N_3 is the number of peripheral bodies; m_i is the initial total mass of the structure; p_{m0} is part of mass m_i, occupied by the central body; A_{Sm} is the semiaxis of the initial orbit in astronomical units (AU); e is the eccentricity of orbits of peripheral bodies; k_φ and $k_{\varphi v}$ are the coefficients of the initial angles of the bodies with the structure construction; and ρ_b is the absolute density of bodies. It should be noted that density ρ_b of the bodies in kg/m^3 is required for calculating their radii. The radii of the bodies are used in the Galactica system during the calculation of their collisions. In the SphDsSt4.dat file, some other parameters are also specified, which are necessary for the Galactica system.

In the SphDsSt4.for program for bodies from $l = 3$ to N_3, the body coordinates and velocities, according to (13)–(15), are recalculated into coordinate system $x_1y_1z_1$ of the first body using nested cycles. In this program, the algorithms for constructing variants III and IV, described below, are used.

Galactica [7, 8] makes it possible to calculate the dynamics of a spherically distributed structure and to study its evolution. Additionally, it is used in this study to finish the structure's creation. As a result of turns, the structure will be created, in which bodies on the sphere will be organized according to strict mathematical law (13)–(15). After their interaction for some time, the bodies are distributed uniformly over the sphere. For this distribution, only the Galactica system is used.

In the input file for the Galactica system, dimensionless quantities are used [7]. All body masses are related to total system mass $m_{ss} = mi$. The time is expressed in hundreds of periods P. For this purpose, the time coefficient is introduced

$$k_t = 1/(100\,P). \tag{16}$$

It should be noted that period P is calculated in the SphDsSt4 program according to (6). Geometric dimensions in the Galactica system are related to quantity

$$A_m = \left(G \cdot m_{ss}/k_t^2\right)^{1/3}. \tag{17}$$

In the Galactica system, differential equations of body interaction by Newton's law of universal gravitation are integrated. In dimensionless form, e.g., for the x projection, they look as follows:

$$\frac{d^2 x_j}{dT^2} = -\sum_{k \neq j}^{N} \frac{m_{ok}(x_j - x_k)}{r_{jk}^3}, \tag{18}$$

where $x_j = x_{C,j}/A_m$ is the dimensionless coordinate of the ith body; $x_{C,j}$ is the coordinate of the ith body relative to the center of mass of the entire structure; $m_{o,k} = m_k / m_{ss}$ is the dimensionless mass of the kth body; $j = k = 1, 2, \ldots, N$; and $N = N_3 + 1$.

In the Galactica system, a high-accuracy integration method is used [7]. In respect to solar system dynamics, the Galactica-system accuracy exceeds the accuracy of NASA programs by orders of magnitude [9]. The Galactica system with a set of necessary means for solving problems is freely available at http://www.ikz.ru/~smulski/GalactcW/. Its description is presented in the file GalDiscrp.pdf in Russian, and in English, GalDiscrpE.pdf. The SphDsSt4 program, data file SphDsSt4.dat, and the mentioned files of structures are presented at http://www.ikz.ru/~smulski//Data/SphDsStr/.

5. VARIANTS OF STRUCTURE CREATION

The above algorithm is the first variant (variant I) of structure creation. This structure with $N_3 = 99$, $k_\varphi = 1.72$, and $k_{\varphi v} = 1$ is presented in Fig. 3 in coordinate system xyz, related to the first body, which was named above as $x_1y_1z_1$. The considered structures had the following dimensional parameters: the central body mass is equal to the Sun's mass, the mass of peripheral bodies is equal to the mass of all planets, while the sphere radius equals the semiaxis of Earth's orbit. A period of revolution of peripheral bodies under these conditions is close to 1 year. As can be seen from Fig. 3, the bodies are located along the circle in the upper hemisphere. The line of their location twice rounded the upper hemisphere and on the second revolution of the body approached the bodies of the first revolution. A drawback of this structure is the fact that the bodies are only in the upper hemisphere, and with successive revolutions, on the line of their disposition, they may be superimposed one on another.

In the second variant (II), the velocity vectors of odd bodies, beginning from the third body, turned downward (Fig. 2), i.e., $\Delta\theta < 0$, while the velocity vectors of even bodies turned upward. A view of structure II with $k_\varphi = 0.8$ is shown in Fig. 3. As can be seen, the velocity vectors of bodies 98 and 99 intersect. Therefore, the paired approaches and collisions of neighboring bodies occur in the studies of the Galactica system of structure interactions. After 100 revolutions of bodies in this structure there were 51 collisions.

In the third variant (III), the velocity vectors of even bodies not only turned upward by angle $\Delta\theta$, but also reversed direction. As can be seen from Fig. 3, in structure (III) with $k_\varphi = 0.8$, the velocity vectors of neighboring bodies, e.g., 98 and 99, are directed apart. Therefore, while interacting, these bodies move apart. In this structure, there was no collisions within 100 revolutions.

The third variant of the algorithm is presented in the SphDsSt4.for program. As already noted, it is based on the sequential rotation of the velocity vectors of bodies of an axisymmetric structure. In this case the bodies are located in the pericenters.

In the fourth variant, each ith body in its orbit is located at its polar angle $\varphi_{0,i}$, according to Eq. (1). Figure 1b shows the position of body m_i in its orbit. Polar radius r_i of the body is defined by Eq. (2). Its radial velocity is [3]

$$v_{r,i} = \pm v_p \sqrt{(\alpha_1 + 1)^2 - (\alpha_1 + R_p/r_i)^2}. \tag{19}$$

The radial velocity is positive when the body moves from the pericenter to the apocenter, and negative,

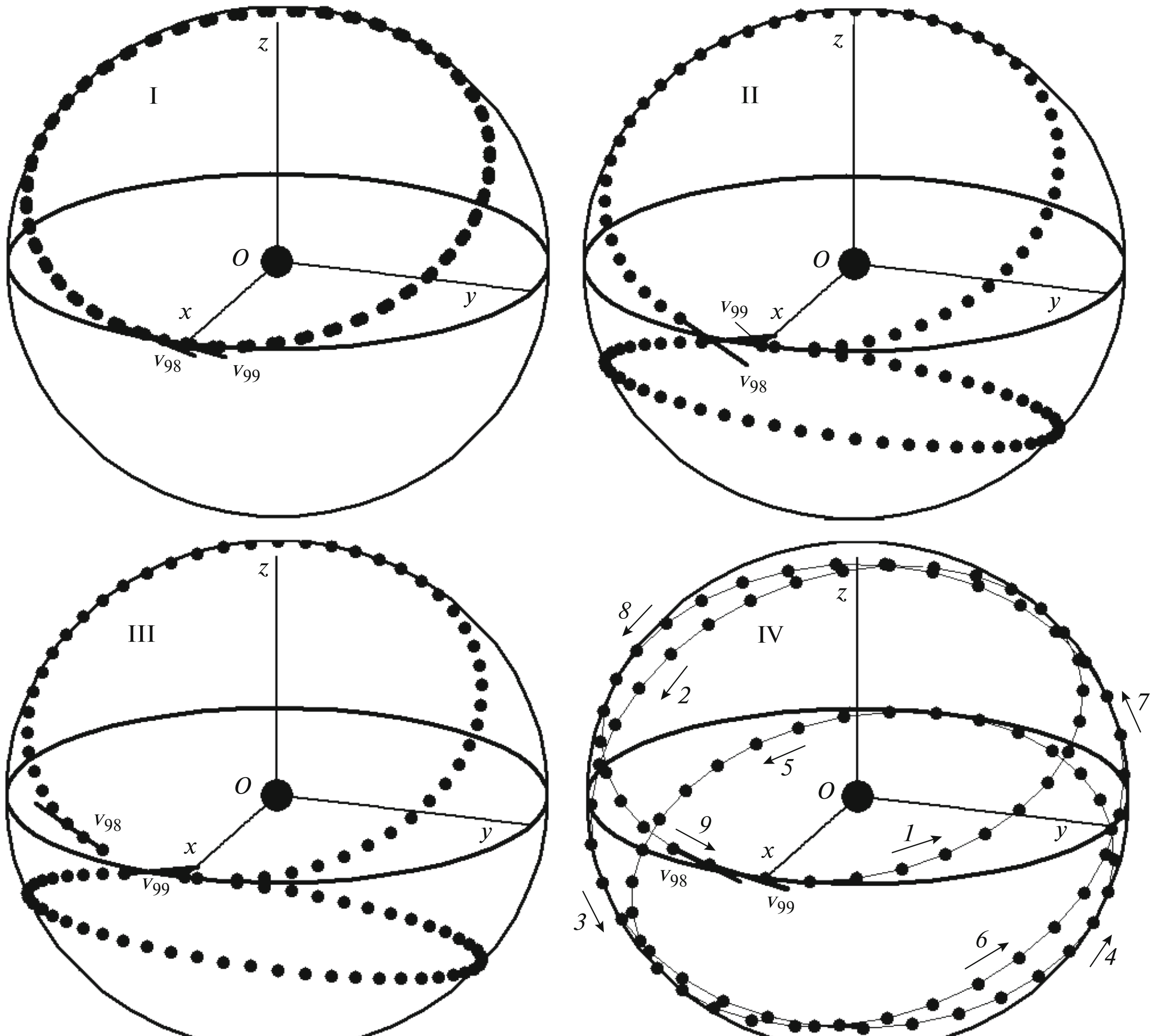

Fig. 3. Four variants (I, II, III, and IV) of creation of the spherically distributed structure with $N_3 = 99$ peripheral bodies (the total structure mass $m_i = 1.99179 \times 10^{30}$ kg; central body mass $m_0 = 1.98912 \times 10^{30}$ kg; peripheral body mass $m_1 = 2.69596 \times 10^{25}$ kg; circular orbit radius $a = 149.598$ million km; and period of revolution $P = 0.99945$ sidereal years). Body m_1 is located on the x axis; the velocity vectors of bodies 98 and 99 are shown with linear segments; arrows 1, 2, ..., 9 show a sequence of location of peripheral bodies in variant IV; a sidereal year is the period of revolution of the Earth around the Sun with respect to fixed stars.

when it returns to the pericenter. A transversal velocity is written in form [3]

$$v_{t,i} = v_p R_p / r_i. \tag{20}$$

Coordinates and velocities of body m_i in coordinate system $x_i y_i z_i$ with the x_i axis, passing through pericenter $P_{c,i}$ (Fig. 1b), will be

$$x_{p,i} = r_i \cos\varphi_{0,i}; \quad y_{p,i} = r_i \sin\varphi_{0,i}; \quad z_{p,i} = 0; \tag{21}$$

$$\begin{gathered} v_{xp,i} = v_{r,i}\cos\varphi_{0,i} - v_{t,i}\sin\varphi_{0,i}; \\ v_{yp,i} = v_{r,i}\sin\varphi_{0,i} + v_{t,i}\cos\varphi_{0,i}; \quad v_{zp,i} = 0. \end{gathered} \tag{22}$$

As mentioned above, in the fourth variant on the initial circle (Figs. 1, 2), there are no bodies m_i, but orbit pericenters $P_{c,i}$ of the bodies m_i. They are spaced on the circle at intervals of $\Delta\psi$. With the orbit turned by angle $\Delta\theta$, the pericenters will be located on the sphere, while proper bodies m_i will be located on their orbits at the angular distance from pericenters $\varphi_{0,i}$. In this case, velocities of bodies m_i also are related to their pericenters by expressions (22). Therefore with orbit turns, coordinates and velocities (21)–(22) will be transformed, according to Eqs. (13)–(15), from the coordinate system of the ith orbit to the $(i-1)$th orbit. The entire algorithm is presented in the SphDsSt4.for program.

By the fourth variant (IV), the structure was created with $N_3 = 99$, $k_\varphi = 1.72$, and $k_{\varphi v} = 1$ (Fig. 3). The bodies along the disposition line on the sphere, beginning with the first one on the x axis, start to bend around the upper hemisphere (arrow *1*) from the front and stop bending at backside *2*. Then, they go to the front of lower hemisphere *3* and pass it below *4*. Next, they go over the front of upper hemisphere *5* and go over the backside of lower hemisphere *6*. Then, they appear on upper hemisphere *7*; at the top, they pass over backside *8*, while at the bottom, they go out on front *9* and thus the position of body 99 ends near the 1st body. So, in this variant of the algorithm, the bodies occupy the sphere along the line that made three turns.

When calculating the dynamics of this structure using the Galactica system, it remained unchanged for ten revolutions. After 100 revolutions, the bodies were distributed uniformly over the sphere. This new property of structure invariability during the ten revolutions is of significant interest: is it possible to create a structure that does not change for a long time?

6. DYNAMICS AND EVOLUTION OF STRUCTURES DISTRIBUTED OVER THE SPHERE

6.1. Different Number of Bodies

The motion of bodies in structures was studied by integrating the differential equations of their motion (18) using the Galactica system. Structures with 2, 11, 99, and 999 peripheral bodies were considered.

In the case of two peripheral bodies, revolving in mutually perpendicular planes, the system was explored for the time of 1000 revolutions. Planes of orbits vary, and the orbital radius changes. After 500 revolutions, a relative deviation of orbital radius δr fluctuates in the 4th digit. Then it grows and, by the 1000th revolution, reaches 18.7%. The orbital radius of the second body at that time decreases by 10.8%.

This three-body system is asymmetric; therefore, changes take place. In a symmetric system, when peripheral bodies are in the same plane, the system exists without change almost for an indefinite time.

The structure of 11 peripheral bodies, created by variant III, remained unchanged within the entire explored interval of 100 revolutions. By the 100th revolution, the largest variations in the radius of orbits did not exceed $\delta r = \pm 0.06$.

6.2. Elliptic Orbits

In the above structures, eccentricity $e = 0$. The structures with elliptic orbits having eccentricity $e = 0.3$ were created. In the process of motion, the structure in variant III increased and decreased in size, i.e., pulsated. By the 100th revolution, the bodies were distributed uniformly over the sphere, and pulsations of the structure size had ceased. In variant IV, all bodies are initially distributed along the elliptic orbit; therefore, this explicit pulsation is absent.

6.3. Different Masses of Bodies

The structures with different masses of peripheral bodies were also considered. In previous structures, a mass fraction of the central body was $p_{m0} = 0.99866$. With an increase in masses of peripheral bodies up to a half of the system mass ($p_{m0} = 0.5$), the structure center-of-mass considerably shifts away from the center of body m_0. Velocities of peripheral bodies at the moment of structure creation diminish from 8.6 to 8 relative units. However, due to large masses of peripheral bodies, the approach of two of them occurs for time $T = 1.4 \times 10^{-4}$ and the initial configuration of the structure is violated. Here, T is the dimensionless time, the unit of which is equal to a "sidereal" century, i.e., 36525.636042 days. By moment $T = 1$, i.e., by 100 revolutions, there were 11 collisions in the structure, and its size increased by 400000 times, i.e., it was destroyed completely.

With the smaller mass of peripheral bodies ($p_{m0} = 0.9$), they were distributed in space for three revolutions; in this case the system size increased not by much. With further motion, the system size remains unchanged up to $T = 0.44$. By 100 revolutions, three collisions occurred in the system, and it considerably increased in size.

In two previous structures with $p_{m0} = 0.5$ and $p_{m0} = 0.9$, the integration step was $\Delta T = 1 \times 10^{-7}$. With a further decrease in the mass of peripheral bodies ($p_{m0} = 0.95$), the structure was more stable, therefore the integration step was $\Delta T = 1 \times 10^{-6}$. In this structure, by 100 revolutions, there were seven collisions; here, one collision was with the central body, while four bodies have spread far beyond the boundaries of the structure.

At the moment of structure creation (Fig. 3), the distances between the bodies are smallest. Then, when bodies are in motion, they are distributed over the sphere, and distances increase. Therefore, with large masses of peripheral bodies, their interaction at first will destroy structures, before the bodies are distributed over the sphere. Therefore, masses of peripheral bodies were increased in the already created structure after 100 revolutions. Coordinates of the new structure remained the same, while velocities were multiplied by coefficient k_{vcr}. This coefficient was defined as a ratio of the peripheral body velocity (in the axisymmetric planar structure with new masses) to the velocity in the structure with previous masses. Thus, the structure with $p_{m0} = 0.5$ was created. Its dynamics was explored with the step $\Delta T = 1 \times 10^{-7}$ up to $T = 0.033$, i.e., for three revolutions of the peripheral body. For this time there was only one collision of the peripheral body with the central body, and the system size increased to some extent. In other words, the stability of this system considerably increased in comparison with the previously considered structure with $p_{m0} = 0.5$.

In the same way, the structure with $p_{m0} = 0.9$ was created. Its motion was considered with integration step $\Delta T = 1 \times 10^{-6}$ for $T = 1$ century. During this time, there was a collision of body 85 with body 73, which did not lead to structure change. However, an approach of body 79 with the central body resulted in considerable changes. This body acquired great velocity and was ejected from the structure. The central body together with several bodies was also ejected from the structure in the opposite direction. The structure with the remaining bodies began increasing in size.

The mechanism of approach of the peripheral body with the central body, with the successive ejection from the structure, is considered in detail by the example of Coulomb interaction of multilayer structures [11–13]. This mechanism is valid also in the case under consideration.

The performed studies with different masses of peripheral bodies testify that with $p_{m0} < 0.95$, to create a spherically distributed structure is problematic with the given structure radius, which in terms of dimensional quantities equals Earth's orbit radius. For a further increase in the mass of peripheral bodies the structure radius should be increased.

Fig. 4. M 53 globular star cluster (or NGC 5024) in the Coma Berenices constellation.

7. DYNAMICS AND EVOLUTION OF THE STRUCTURE OF 1000 BODIES

Main studies of structure dynamics and evolution were performed with the total number of bodies equal to 1000, which nears the number of bodies in globular star clusters. As an example, Figure 4 shows the M53 globular star cluster in the Coma Berenices constellation (https://ru.wikipedia.org/wiki/M_53). It is located at a distance of 60000 light-years from the galactic center and almost at the same distance from the solar system.

In central regions of globular star clusters, the star density is 100–1000 stars per cubic parsec (1 pc = 206264.8 AU), whereas in the vicinities of the Sun, it is around 0.13 $1/\text{pc}^3$, i.e., by 700–7000 times less. Diameters of clusters are 20–60 pc, while their masses are 10^4–10^6 of the Sun's mass. In the center of globular star clusters, there are massive stars, with a mass of approximately 10^4 and more of the Sun's mass. Due to a high density of stars, the close passes of stars and their collisions frequently take place in clusters. With a large magnification, as can be seen in Fig. 4, a globular star cluster has no clear boundary and is a structure with a gradual decrease in the number of stars with distance away from its center.

A structure was created using variant III with $N_3 = 999$ and parameters presented in Fig. 5 (file StrD999c.dat). With this number of bodies, the distances between them are small. For the first bodies, planes of orbits also differ little, while velocity vectors of neighboring bodies are oppositely directed. This leads them into a collision. These problems were computed with step $\Delta T = 1 \times 10^{-7}$. After 2000 steps, the collision of body 2 with body 10 occurred. After the removal of body 2 and new structure creation, this situation was repeated for body 4. Thus, by removing bodies 2, 4, 8, 10, and 15 from the StrD999c.dat structure, the StrD994c.dat structure was created. The motion of bodies of this structure was studied by the Galactica system with step $\Delta T = 1 \times 10^{-7}$.

As the bodies moved, their initial positions on two circles (Fig. 5) changed, and the bodies were distributed more uniformly over the sphere. In this case, collisions occurred between individual peripheral bodies, and they united into a double-mass body. One more body may join the double-mass body, and the triple-mass body will be obtained. Additionally, the peripheral body may collide with the central body. Table 1 presents the collision history for the explored period $T = 1.65$ century.

As can be seen from Table 1, the main collision number $k_{imp} = 40$ occurred for the first five revolutions, which amounts to 2 out of 3 collisions for the entire time interval including 165 revolutions. In this case, 32 double-mass bodies and 2 triple-mass bodies were created. Here, four peripheral bodies collided with the central body. For an accurate determination the bodies with which masses formed the triple-mass body and which collided with the central body, it is necessary to carry out a logical analysis of these results. In this case it is necessary to keep in mind that

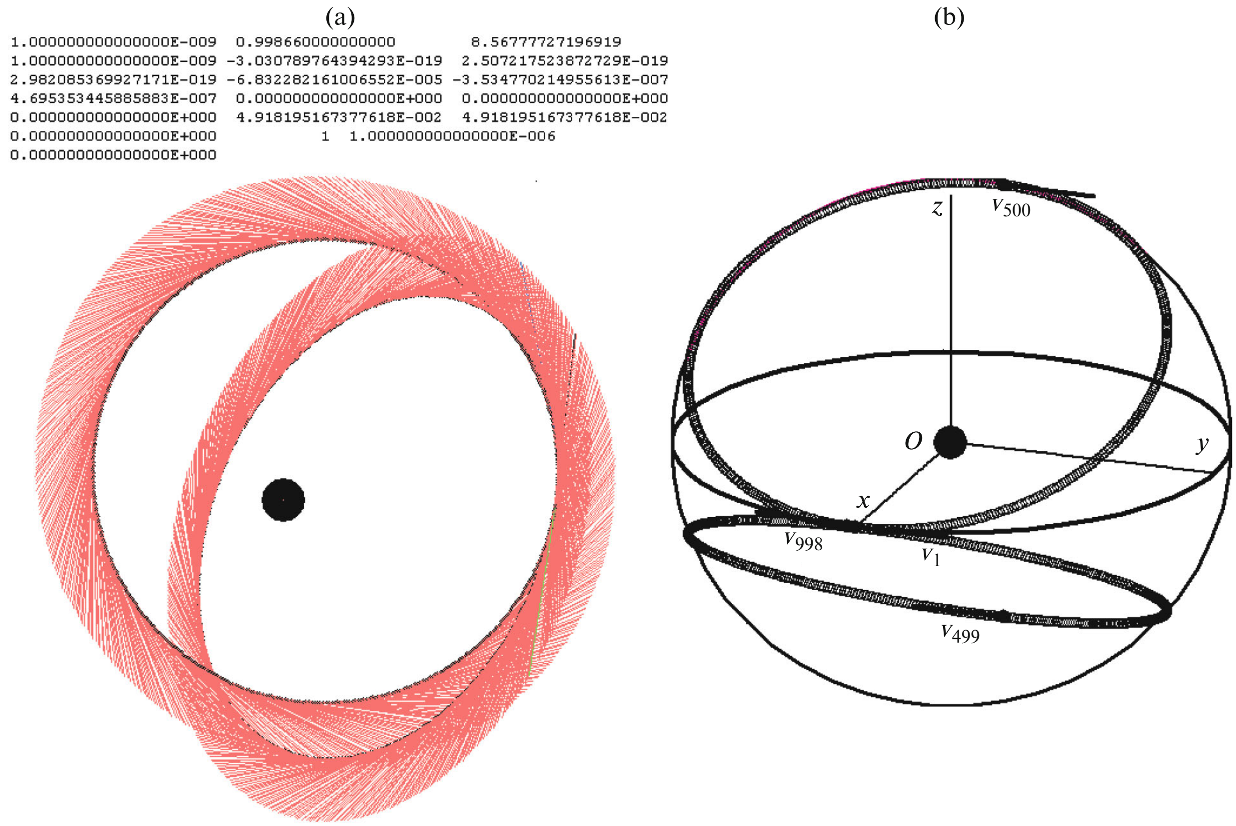

Fig. 5. Spherically distributed structure by variant III with $N = 1000$, $k_\varphi = 0.86$, $k_{\varphi v} = 1$, $R = 1$ AU, $P_{rd} = 1$ year and with the central body mass equal to the Sun mass: (a) projection onto a horizontal plane after the structure-motion calculation by the Galactica system for a single time step (numerals show dynamical parameters of the structure and characteristics of computational process); (b) view of the frontal plane in the coordinate system *xyz* (lines near bodies 1, 499, 500, and 998 show the velocity vectors).

in the Galactica system, when two bodies collide, the number of the one which had a larger mass is assigned to the created body, while the mass of the one which had a smaller mass is put to zero. Therefore, here, collision number k_{imp} is determined by the number of bodies with zero mass.

Over the next 5 years, as can be seen from Table 1, five collisions occurred. In this case, a single triple-mass body created. For 5 years, from 10 to 15 revolutions, there was a single collision; from 15 to 20 revolutions, three collisions occurred, while for 40 years, from 20 to 60 revolutions, there were no collisions. Collisions were also absent for the 10-year gap from 70 to 80 and from 90 to 95 revolutions. For the remaining time intervals, the collisions occurred by ones and twos.

The structure created after 100 revolutions, with a distribution of bodies over the entire sphere, is shown in Fig. 6. The distances of peripheral bodies from the central body predominantly differ from the initial distance by no more than 3%. Their periods of revolution are close to 1 year. No bodies were ejected from the structure. A single body alone, namely body 2, acquired a

Table 1. Dynamics of collisions of bodies of the spherically distributed structure StrD994c.dat for 165 revolutions: k_{imp} is the collision number; k_{2m1} is the number of double-mass bodies; k_{3m1} is the number of triple-mass bodies; k_{m0} is the number of bodies collided with the central body

Parameter	Parameter change in time T														
T, years	5	10	15	20	60	65	70	80	90	95	100	105	125	150	165
k_{imp}	40	45	46	49	49	51	54	54	55	55	56	57	58	59	61
k_{2m1}	32	35	34	37	37	39	40	40	41	41	40	41	42	43	45
k_{3m1}	2	3	4	4	4	4	4	4	4	4	5	5	5	5	5
k_{m0}	4	4	4	4	4	4	6	6	6	6	6	6	6	6	6

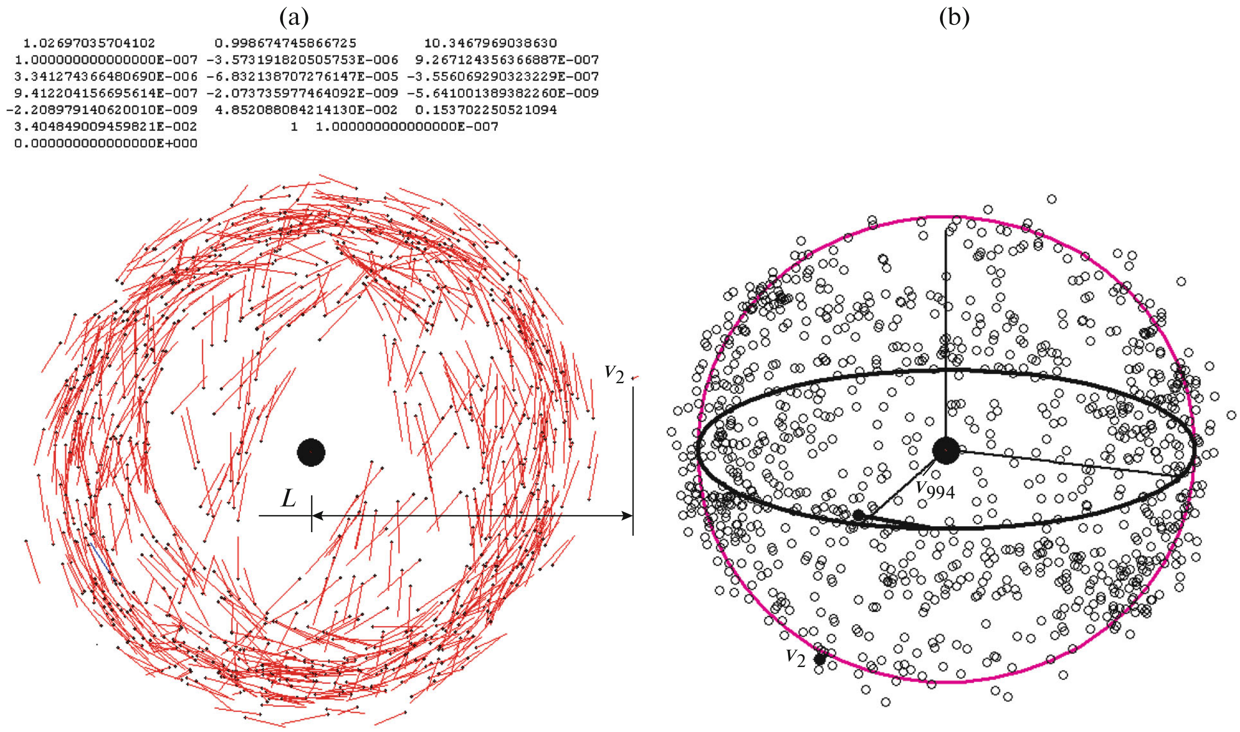

Fig. 6. Spherically distributed structure StrD994c.dat with N_3 = 994 after motion for time T = 1.027 century. Distance L to body 2 is reduced by half in the figure. A period of revolution of body 2 is close to 1 year, (a) Projection onto a horizontal plane with computation using the Galactica program. (b) View of the frontal plane in the coordinate system xyz (velocity vectors of bodies 2 and 994 are shown with linear segments).

great ellipticity of orbit, but its period of revolution did not change considerably. The apocenter of its orbit is at distance L = 2.8 radii of the structure (in Fig. 6a, distance L to body 2 is reduced by half).

The body distribution over the sphere, presented in Fig. 6, already has formed by the 20th revolution. Body 2 went beyond the boundaries of the initial sphere for the first 5 revolutions. Therefore, further development of the structure up to the 165th revolution can be considered already as a stable existence of the spherically distributed structure. At times, collisions of bodies occur, and they unite into a single body. However, the longer the structure lives, the lower the number of these collisions. In this structure, bodies with neighboring numbers initially orbit in opposite directions (see Fig. 5). However, there are no collisions at counter velocities. All collisions occur during the same-direction approach in the intersecting orbits of bodies. Solely for body 2, probably, the approach occurred at counter velocities, and, due to a change in the velocity vector, it approached the central body. After interaction with the central body, it transferred to the highly elliptical orbit.

The dynamics and evolution of two more structures were considered. The St4D999d.dat structure with 999 peripheral bodies was created according to variant IV with coefficient k_φ = 1.72. It was computed for T = 1.9 century with step $\Delta T = 1 \times 10^{-7}$. With this coefficient k_φ, distances between the bodies are identical, except for bodies 999 and 1: the distance between them is 4 times larger. By the end of five revolutions, the bodies were distributed over the sphere, and there were k_{imp} = 158 collisions, which have led to the creation of bodies with double mass (k_{2m1} = 141), triple mass (k_{3m1} = 1), and quad mass (k_{4m1} = 5). By the end of ten revolutions, parameters of collisions were: k_{imp} = 160; k_{2m1} = 140; k_{3m1} = 1; k_{4m1} = 6. By the end of 100 revolutions, these parameters changed insignificantly: k_{imp} = 171; k_{2m1} = 139; k_{3m1} = 3; k_{4m1} = 7; k_{6m1} = 1. Even smaller changes occurred for the successive 90 revolutions to T = 1.9 century: k_{imp} = 173; k_{2m1} = 139; k_{3m1} = 3; k_{4m1} = 6, and k_{6m1} = 2.

In this structure after the 70th revolution, the body with $6m_1$ appeared, and there were no collisions with the central body. In Fig. 7b, this structure is shown for comparison with the previous structure (Fig. 7a) after 100 revolutions. As noted above, the structure (Fig. 7b) formed during five revolutions, i.e., well before the previous one (Fig. 7a). In it, there were no collisions with the central body and there were no large ejections of the peripheral body.

The St4D999e.dat structure was created according to variant IV with further refinement. Coefficient k_φ is selected such that the distance between bodies 999 and 1 could coincide with the remaining distances between

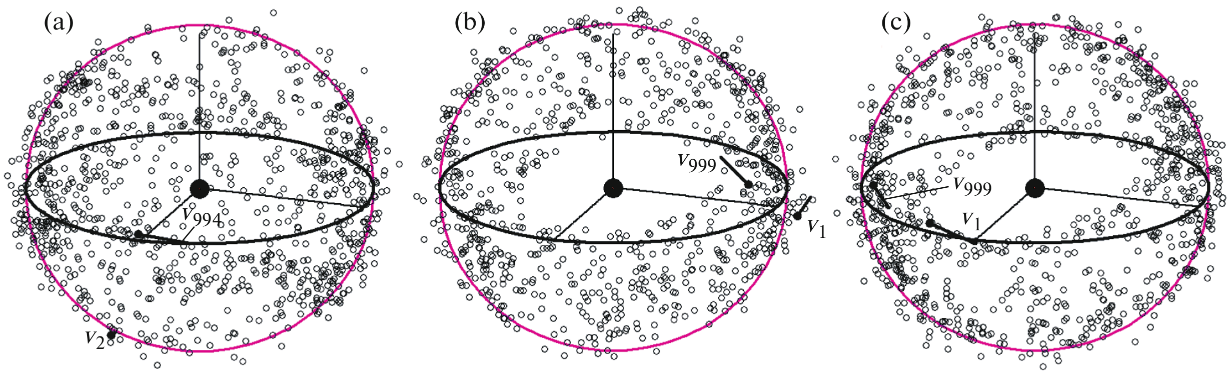

Fig. 7. Spherically distributed structures with $N \approx 1000$, initially created by different variants, after 100 revolutions of peripheral bodies: (a) variant III (see Fig. 6); (b) variant IV; (c) variant IV with refinement of a distance between bodies *1* and N_3.

bodies. We will call this structure regular (Fig. 7c). It is more stable than previous ones. By the end of five revolutions, more than half the bodies were located on the line of their initial creation. Nevertheless, by the end of ten revolutions, all bodies were uniformly distributed over the sphere. The number of collisions in this structure was an order of magnitude less. For example, by the end of 5 revolutions, $k_{imp} = 14$ and $k_{2m1} = 14$; by the end of 100 revolutions, $k_{imp} = 26$, $k_{2m1} = 24$, and $k_{3m1} = 1$. In this case, from revolution 95 to revolution 160, there were no new collisions. Thus, the initial regular form of structure creation subsequently leads to the more stable structure with the uniform distribution of all bodies over the sphere.

It should be noted that in the steady structures, all bodies are in periodic motion. The period of motions is the same as in the initial planar structure from which the spherical structure was created. Even the period of body 2 in Fig. 6 (with the highly elliptical orbit) almost has not changed.

Comparing the shape of structures obtained (Fig. 7) with the image of the globular star cluster in Fig. 4, the following can be noted. Bodies in the structures are located more compactly than in the globular star cluster. It is explained by the "manmade" nature of the structure formation: its initial organization reduces several close encounters, which subsequently lead to the larger spread of bodies over the space.

There are also differences between the structure and globular star cluster, which are caused by the structure model itself: it is the incomplete central symmetry of the structure and its sparseness in the central region. In this case, in the less regular structure by variant III (Fig. 7a), the central region is more filled. The concentration of bodies at the center can be increased by creating multilayer spherically distributed structures. In multilayer structures, the mass fraction of central body p_{m0} can be reduced. For example, in planar 15-layer structures [5, 6], it amounts to $p_{m0} = 0.006$.

As far as evolutions of the structure and globular star cluster are concerned, then they are similar. These objects can exist for a long time without change. At times, approaches of bodies may occur, which will lead to a change in their orbital radii or to their collision. The longer the structure exists, the smaller the number of these approaches. This is proven by properties of globular star clusters, which were noted above.

8. SCALING THE STRUCTURES

As mentioned above, in the SphDsSt4 program, files of structures with initial conditions, e.g., St4D999e.dat, using parameters m_{ss}, A_m, and k_t, are created in dimensionless form [7]. According to (17) these parameters are related to one another; therefore, with known A_m and m_{ss}, the time coefficient is defined as

$$k_t = \sqrt{Gm_{ss} / A_m^3}. \quad (23)$$

In the St4D999e.dat structure with $N = 1000$ bodies, system mass $m_{ss0} = 1.99179 \times 10^{30}$ kg, central body fraction $p_{m0} = 0.99866$, orbit semiaxis $a_0 = 1$ AU, scale parameters were $A_{m0} = 1.0973762 \times 10^{13}$ m and $k_{t0} = 3.1687536 \times 10^{-10}$ 1/s. The dimensionless semiaxis was $a_{un} = 0.0136323$ and the dimensionless period of revolution of the peripheral body was $P_{un} = 9.9920079 \times 10^{-3}$.

We consider another structure with parameters close to the globular star cluster: $m_{ss} = 10^5 M_s = 1.989118 \times 10^{35}$ kg, where M_s is the Sun's mass; the orbit semiaxis $a = 1$ pc $= 206264.8$ AU $= 3.0856775 \times 10^{16}$ m. Let us find parameters at which the initial conditions in the file St4D999e.dat with dimensionless semiaxis a_{un} and period P_{un} will correspond to this structure. The new scale parameter is $A_m = a/a_{un} = 2.263501 \times 10^{18}$ m. Time coefficient $k_t = 1.06980958 \times 10^{-15}$ 1/s is determined depending on new m_{ss} and A_m from Eq. (23).

All results, including Fig. 7, with integration of differential equations of motion by the Galactica system with initial conditions of the file St4D999e.dat correspond to the new structure with parameters A_m and k_t. For example, the period of revolution of peripheral bodies in it

$$P = P_{un}/k_t = 9.34 \times 10^{12} \text{ s} = 296.198 \text{ thousand years.} \quad (24)$$

In other words, if in the previous structure the peripheral bodies revolved around the central body for 1 year, then in the new structure, it will for almost for 300000 years.

In the example, we considered a structure with a new mass and a new size $2a = 2$ pc. The results can also be used to study the influence of the structure mass or its size on its characteristics, if only the mass or size of the system is varied. Therefore, the scaling algorithm given here can be used to study gravitational interactions in the structure in a variety of ways based on the above derived solutions.

9. FURTHER STUDIES

The globular star clusters obtained in this problem have a massive core and most stars are located in the peripheral region. In the Universe, the most diverse globular star clusters are observed. They are systemized using different properties, e.g., according to degree of star concentration toward the center, the globular star clusters are divided into 12 classes: I, II, ... XII, where I refers to a cluster most concentrated toward the center [1]. Probably, the globular star clusters obtained here can be referred to as later classes.

The results require understanding and further development. For example, periods of revolution of bodies in the planar axisymmetric structure and in the spherically distributed structure coincide. This indicates that the forces of all bodies on one of the peripheral bodies coincide in these two structures. This result is obtained based on numerical calculations. It is necessary to prove it theoretically.

Using the algorithms obtained, it seems interesting to deploy a multilayer rotating structure in space [5, 6]. This structure will simulate a globular star cluster filled with bodies not only over the sphere but also inside it, i.e., more closely to class I. In this case, bodies on all layers have the same period of revolution. Will it remain unchanged? Can regular motions of bodies be created in layers?

The globular star clusters, more concentrated toward the cluster center, i.e., more closely to class I, can be created by successive combining spherically distributed structures by the type of the "nesting dolls": the first structure is a central body for the second structure; the second one together with the first structure is the central body for the third structure, and so forth. With the uniform distribution of bodies over the sphere, their action on the structure, located at the center, tends to zero. Therefore, the evolution of the central structure will not be violated by action of bodies of the outer shell. This statement shows the way for creating this volume-filled spherical structure.

The developed algorithm can be applied for creating the spherically-distributed Coulomb structures [11–13]. Probably, creating and studying them will bring us to understanding the structure of an atom.

With the statistical approach to the evolution of associations of stars for creating a function of phase space density, it is necessary to create a set of orbits (orbit library), in order to achieve required potential Φ by combining them. This library of orbits can be filled using the method of turning orbits in space, which is considered in this study.

CONCLUSIONS

As a result of multiple interactions spherically organized structures with periodic motion of bodies can be created. Despite the fact that orbits of bodies are located in different planes, these structures can exist forever. In the process of formation of the structures near-collisions occur at which ejections of bodies from the structure are possible. When bodies collide of bodies, they merge. However, the longer the structure exists, the less these phenomena occurrs in it. With time, in this cluster, the bodies remain with orbits in which no dangerous approaches and collisions of bodies occur, while if they happen, they happen very seldom.

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