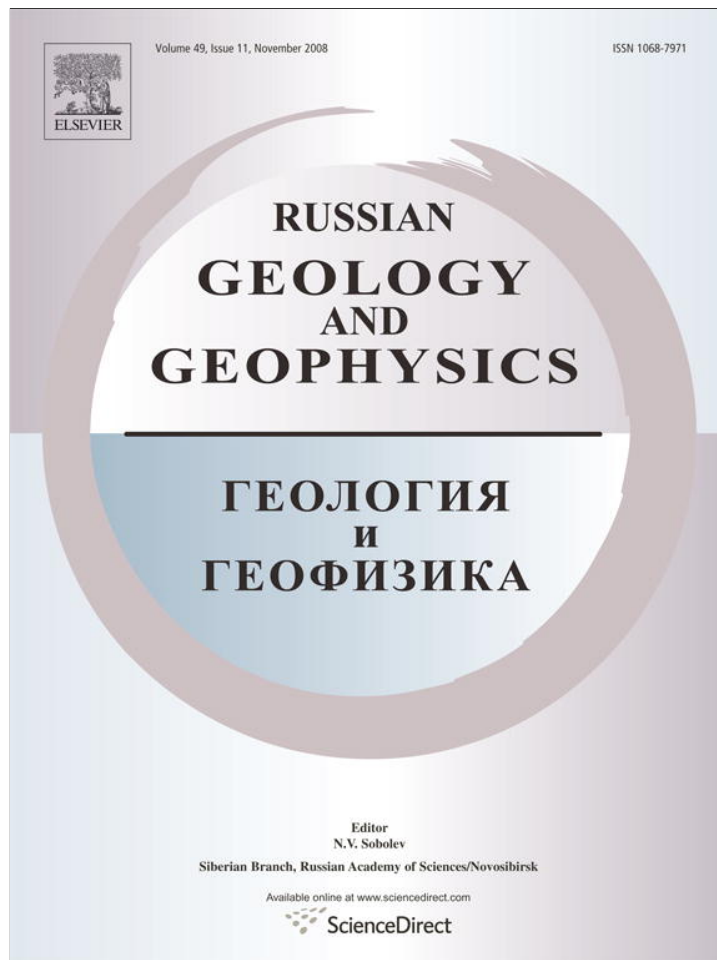


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Compound modeling of Earth rotation and possible implications for interaction of continents

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Received 28 March 2007; in revised form 11 March 2008; accepted 21 April 2008

Abstract

The theory of orbital climate forcing implies an approximate Earth rotation model which has never been checked against other methods. We simulate the rotating Earth in compound models as a system of equal peripheral parts that orbit a central body and investigate the orbital evolution as affected by the gravity pull from the Sun, the Moon, and the planets. The predicted nutation cycles agree well with estimates by other methods. The model of periodic convergence and divergence of peripheral bodies may be useful to explain interaction of continents. © 2008, IGM, Siberian Branch of the RAS. Published by Elsevier B.V. All rights reserved.

Keywords: Earth rotation; modeling; nutation; interaction of continents

Introduction and problem formulation

Modeling is a commonly used tool to look into complex phenomena which are too hard to investigate otherwise. Such are many global or astronomical issues, for instance, the formation of protoplanets simulated in (Serbulenko, 1996), or the behavior of the Earth's spin axis over long periods of time which is a challenging problem with many unknowns. We address the dynamics of a rotating Earth to reconstruct the evolution of its axis.

The amount of heat the Earth receives from the Sun depends on its inclination relative to the incoming sunlight, insolation time, and distance from the Sun. The position of the Earth's surface with respect to the Sun is controlled by the position of the Earth's orbit and spin axis changing under the effect of the planets, the Moon, and the Sun. According to Milankovitch's theory (1930), the Earth's surface insolation at different latitudes depends on the orbital parameters (Fig. 1): eccentricity, pericenter longitude $\varphi_{p\gamma} = \gamma B$ (position relative to the moving equatorial plane), and especially on obliquity ε (tilt of the Earth's spin axis relative to its orbital plane). The latter oscillates at period $T_\varepsilon = 41.1$ kyr, which corresponds to the major insolation cycle (Berger and Loutre, 1991). The

obliquity cycle was derived from approximate analytical equations of motion for an orbiting and rotating Earth.

Ever more precise solutions of the orbital motion equations have been a subject of special recent research, and reliable results are available for the evolution over 100 Myr. However, the previous solutions of the Earth rotation problem were schematic: Second-order differential equations were reduced to first-order Poisson equations and the obtained estimate of the obliquity cycle ($T_\varepsilon = 41.1$ kyr) has never been checked against other solutions. In this respect, there are some questionable points. First, they are only second-order differential equations that can describe real rotational motion, and recent numerical solutions of these equations (e.g., Smul'skii and Sechenov, 2007) over kyr-scale time intervals gave nutation cycles consistent with observed data. Second, the tilt of the equatorial plane relative to the moving orbital plane is unstable and subject to random change in first-order Poisson solutions over long time intervals (Laskar, 1996; Laskar et al., 2005), which supports our doubt on the reliability of the equations themselves. Third, the obliquity ε found from Poisson equations differs from that reported by ancient astronomers (Newton, 1977).

The doubts on the available knowledge of ε evolution and the complexity of the Earth's rotation problem call for alternative solutions. One such alternative is the subject of this study.

Having explored the orbital motion and its evolution for the Earth, the Moon and the planets of the Solar System over

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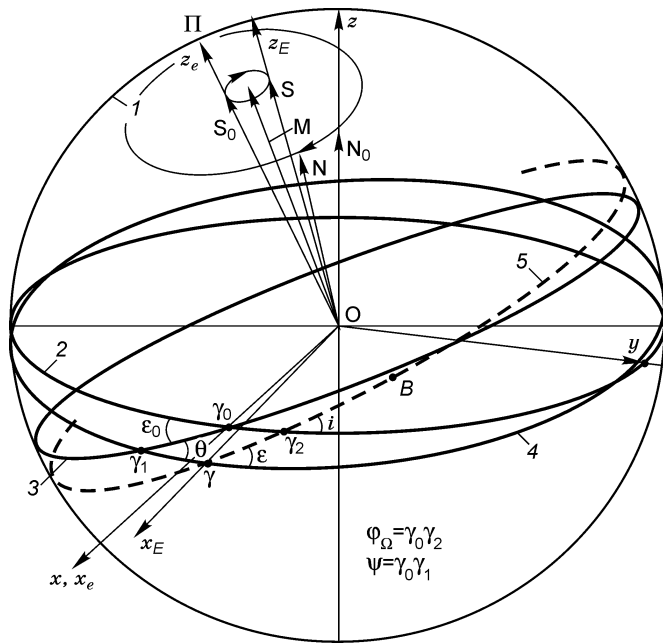


Fig. 1. Earth's orbital elements and axes in fixed equatorial (xyz) and ecliptic ($x_ey_ez_e$) coordinates. 1 — celestial sphere; 2 — Earth's equatorial plane at T_0 ; 3 — Earth's orbital plane (fixed ecliptic plane) at T_0 ; 4 — Earth's equatorial plane at T ; 5 — Earth's orbital plane (fixed ecliptic plane) at T ; 5 — Earth's orbit relative to Sun and orbit of a peripheral body relative to Earth. Letters denote unit vectors: \mathbf{N} — Earth's spin axis and orbital axis of a peripheral body; \mathbf{S} — Earth's orbital axis; \mathbf{M} — angular momentum of Solar System; γ_0 — vernal equinox at T_0 ; B — pericenter longitude; $\varphi_\Omega = \gamma_0\gamma_2$ — angular distance of orbital ascending node; $\varphi_p = \gamma_2B$ — angular distance of pericenter; i — inclination of orbital plane with respect to fixed equatorial plane; $\varphi_{p\gamma} = \gamma B$ — angular position of pericenter in moving coordinates.

100 Myr (Mel'nikov et al. 2000; Mel'nikov and Smul'skii, 2004; Smul'skii, 2005), we obtained that the dynamics of the Earth's orbital plane (5 in Fig. 1) is controlled by the precession of the Earth's orbital axis \mathbf{S} relative to the angular momentum \mathbf{M} of the Solar System with period $T_{0E} = 68.7$ kyr (Smul'skii, 2003a, 2005). The axis \mathbf{S} rotates clockwise about \mathbf{M} , i.e., counter the orbital motion, while the angle between \mathbf{S} and \mathbf{M} (nutation angle θ_S) varies from 0° to 2.9° at different periods, being the greatest for the Earth's orbital axis $T_{sn1} = 97.4$ kyr.

According to astronomic evidence (Duboshin, 1976), the Earth's spin axis \mathbf{N} (Fig. 1) precesses, also clockwise, about the axis \mathbf{S} , with period $T_{prE} = 25.7$ kyr, and the angle ε between \mathbf{N} and \mathbf{S} oscillates with different periods and amplitudes. Thus, the Earth's orbital axis \mathbf{S} , as well as the Earth's rotation axis \mathbf{N} , are subject to both nutation and precession, but unlike \mathbf{N} , the axis \mathbf{S} rotates about the vector \mathbf{M} fixed in the space.

The same computation was applied to the Moon's orbit for time intervals of $-2 \text{ Myr} < T < 0$ and $-100 \text{ Myr} < T < -98 \text{ Myr}$ and showed that the lunar orbital axis precesses about the moving terrestrial orbital axis \mathbf{S} at $T = 18.6$ years and has a minor nutation with its cycle $T_n = 0.47$ year much shorter than the precession cycle. Thus, as it turned out, the behavior of

the lunar orbital axis is very similar to that of the terrestrial spin axis \mathbf{N} , as both precess around the moving axis \mathbf{S} .

Inasmuch as the orbital axes of the planets and the Moon have the same dynamics as the Earth's rotation axis, it is possible to formulate the Earth's rotation problem by simulating a rotating Earth by several axisymmetrical bodies in an equatorial plane. Their positions and velocities can be set precisely with reference to the available exact solution for interaction of such bodies (Grebenikov, 1998; Smul'skii, 1999, 2003b). The system will evolve under the effect of the planets, the Sun, and the Moon, and the behavior of the compound model of a rotating Earth will simulate the evolution of the Earth's spin axis.

Formulation of a compound model of Earth rotation

The rotating Earth is simulated as a system of n bodies with the equal masses m_1 positioned axisymmetrically in the equatorial plane around a central body of the mass m_0 (Fig. 2). The parameters of the compound model are found as follows.

The total mass of the peripheral bodies and the central body equals the Earth's mass M_E :

$$M_E = m_1n + m_0. \quad (1)$$

The peripheral bodies move along a circular orbit at the angular velocity of the Earth's rotation ω_E , which, according to the exact solution of the axisymmetrical problem (Smul'skii, 2003b), is given by

$$G(m_0 + m_1f_n)/a^3 = \omega_E^2, \quad (2)$$

where G is the acceleration due to gravity; a is the orbital radius of n bodies, and the function f_n depends on the number of bodies n :

$$f_n = 025 \sum_{i=2}^n \frac{1}{\sin [(i-1)\pi/n]}. \quad (3)$$

The Earth's moments of inertia relative to the axis x (in the equatorial plane) and to the system of the bodies are equal being

$$J_{Ex} = 0.4M_ER_{Ep}^2 = 0.4m_0R_0^2 + 0.5m_1na^2, \quad (4)$$

where R_{Ep} is the Earth's polar radius and R_0 is the radius of the central body.

Equal are also their inertia moments with respect to the axis z :

$$J_{Ez} = J_{Ex}/(1 - E_d) = 0.4m_0R_0^2 + m_1na^2, \quad (5)$$

where $E_d = 3.2737752 \cdot 10^{-3}$ is the Earth's dynamic ellipticity.

The radiuses of the bodies R_0 and R_1 are inferred from their masses and mean densities as

$$R_0 = (3m_0/(4\pi\rho_E))^{1/3}, \quad R_1 = (3m_1/(4\pi\rho_E))^{1/3}, \quad (6)$$

where the Earth's mean density is

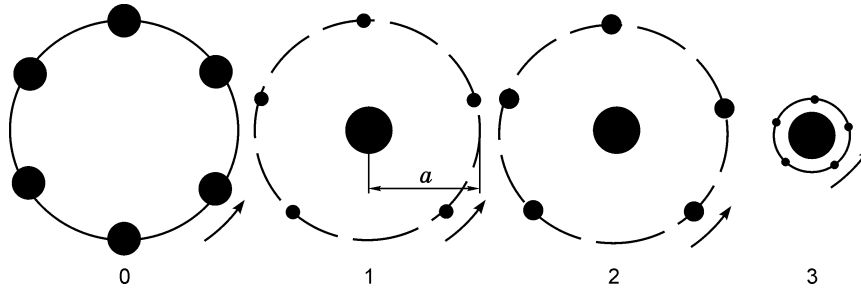


Fig. 2. Models of Earth rotation and model parameters at $R_{Ee} = 6.37816 \cdot 10^6$ m, $M_E = 5.9742 \cdot 10^{24}$ kg; $T_{\pi} = 2\pi/\omega$ is orbital cycle of a peripheral body. Model 0: $m_1 \cdot 10^{-19} = 9.96 \cdot 10^7$ kg, $a/R_{Ee} = 4.45$, $T_{\pi} = 23.93$ hr; model 1: $m_1 \cdot 10^{-19} = 3.57$ kg, $a/R_{Ee} = 6$, $T_{\pi} = 23.93$ hr; model 2: $m_1 \cdot 10^{-19} = 7.21$ kg, $a/R_{Ee} = 6$, $T_{\pi} = 23.93$ hr; model 3: $m_1 \cdot 10^{-19} = 0.54$ kg, $a/R_{Ee} = 1$, $T_{\pi} = 1.408$ hr.

$$\rho_E = 3M_E/4\pi R_{Ee}^3 \quad (7)$$

R_{Ee} being the Earth's equatorial radius.

The system of four nonlinear algebraic equations (1), (2), (4), and (5) defines four elements of the model: the masses m_0 and m_1 , the number n , and the orbital radius a . The system was used to analyze the parameters in wide ranges, including different n . However, there are two points of difficulty: (i) the problem has no solution at some parameter values, and (ii) the equations cannot fulfill exactly because the number of bodies n is discrete. We chose a reasonable number of peripheral bodies and used the following conditions instead of (4) and (5). In model 1, the moment of inertia produced by the peripheral bodies was assumed equal to the difference of the Earth's moments of inertia:

$$m_1 n a^2 = J_{Ez} - J_{Ex} \quad (8)$$

In model 2 we equated the model moments of inertia given by (4) and (5) with that of the Earth to obtain

$$\frac{0.5m_1 n a^2}{0.4m_0^{5/3} (3/(4\pi\rho_E))^{2/3} + m_1 n a^2} = \frac{J_{Ez} - J_{Ex}}{J_z} = E_d \quad (9)$$

We integrated numerically the equations of motion (Mel'nikov and Smul'skii, 2004) of five bodies in the compound model, the Moon, the nine planets of the Solar System, and the Sun, i.e., sixteen bodies altogether, and investigated the evolution of the two models (1 and 2). The resulting precession cycle $T_{pr} = 170$ years of the bodies that orbit the axis \mathbf{S} turned to be times smaller than the precession of the Earth's axis \mathbf{N} ($T_{prE} = 25.7$ kyr). It was, however, possible to increase the precession by reducing the model radius. Thus, in model 3 the radius of the system was assumed to equal the Earth's equatorial radius $a = R_{Ee}$ and the bodies' densities were set twice the Earth's density to avoid collisions:

$$\rho_0 = 2\rho_E \quad (10)$$

The angular velocity ω of an orbiting Earth was found from (2):

$$\omega^2 = G \cdot (m_0 + m_1 f_n) / R_E \quad (11)$$

In addition to the three models of a rotating Earth, we tried another one we called a zero model (Fig. 2). Its parameters were found from (1)–(5) but at $m_0 = 0$, i.e., without the central body. Integration of the equations of motion predicted that the bodies would go off the axial symmetry after two revolutions and then converge rapidly to let the system collapse. The reason is that the large mass of the peripheral bodies causes large gravitation, which increases very fast when the system loses its balance and the bodies converge by the pull from the Moon and other celestial bodies. Therefore, we refused that model and accepted the three others (Fig. 2).

The equations of motion for sixteen bodies were integrated numerically at $\Delta t = 10^{-5}$ years for models 1 and 2 and at $\Delta t = 10^{-6}$ years for model 3. The integration results were regularly saved in files, and the files were used to repeat the calculation for an orbital cycle and 12 orbital elements of each peripheral body relative to the Earth's center (5 in Fig. 1), including the ascending node longitude φ_{Ω} in the fixed equatorial plane, the inclination of the orbital plane i with respect to the equatorial plane, and the orbital eccentricity e .

The calculation was performed for points at different time steps and over different time intervals up to 110 kyr, or continuously through different quantities of peripheral body revolutions, up to 20,000.

Results

Modeling bodies' dynamics. Visualization of the integration results in various modes offered by the *Galactica* software facilitates understanding the mechanism of the bodies' motion and choosing further steps of numerical experiments. Figure 3 shows nine positions of the peripheral bodies with respect to the central body in model 3. The bodies move axisymmetrically before $t = 0.328$ year but after 0.339 year the symmetry upsets and the bodies come into cyclic convergence-divergence motion: they converge the closest at 0.361 year and then diverge to meet again at 0.094 year, and so on.

Similar cycles appear in other models as well. The peripheral bodies in model 1 converge successively at 1.4, 2, 2.6, 3.2, 3.8, 4.7, 5, 5.6, 6.4, 7.1, 7.7, 8.2, and 8.9 years, i.e., the cycle is from 0.5 to 0.8 year, or 0.66 on average over

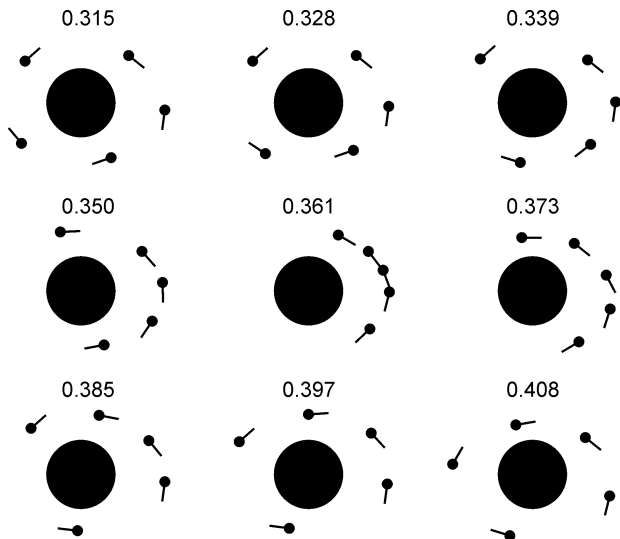


Fig. 3. Dynamics of peripheral bodies in model 3 at different times (in years) since T_0 .

15 years; in model 2 this average is 0.32 year. Model 2 assumes a twice larger mass of the peripheral bodies (Fig. 2) which causes a four times greater gravitation and may be also responsible for the twice shorter convergence cycle. In model 3, the peripheral bodies are spaced six times more closely and their rotations are 17 times more frequent, thus making the convergence cycle still shorter.

Although the bodies converge repeatedly, they never approach the critical distances that would bring the system to collapse. We integrated the equations of motion for the 110 kyr time interval in model 1 and 40 kyr in model 3 but found no signs of possible collapse.

Orbital evolution of a peripheral body. After visual examination of the bodies' dynamics, we focused on the orbits of peripheral bodies relative to the central body. See Fig. 4 for the evolution of three orbital elements (e , φ_Ω , i) in model 1 for 0...–0.5, 0...–5, and 0...–50 years. The angles φ_Ω and i were considered in fixed equatorial coordinates with the central body at the origin O (Fig. 1). The orbits of the peripheral body were modeled continuously for all revolution series rather than over time intervals.

The initial conditions were set for a circular orbit but the eccentricity (e) was nonzero, though very small, already in the first rotation (1 in Fig. 4), i.e., the orbit was not perfectly circular. Then the eccentricity increased in a nonmonotonic way, first with period $T_1 = 0.0714$ year = 26.1 days, while the tendency of time-dependent eccentricity growth extended to longer intervals (0...–5 and 0...–50 years). See well pronounced T_{e1} cycles at the 0...–5 years interval. Another harmonic appeared in the last interval, with period $T_2 = 10$ years, and e reached $6 \cdot 10^{-4}$.

Judging by the ascending node $\varphi_\Omega = 4.3$ rad $\approx 1.5\pi$ (1 in Fig. 4), the orbit rotated about the y axis. At the beginning φ_Ω varied aperiodically in a damped mode, and then grew monotonously through 0...–5 and 0...–50 years.

The inclination i increased continuously through all intervals (Fig. 4) and then decreased again to zero having reached the maximum $i_{\max} = 0.815$ rad (Fig. 5), at $T = 170$ year cycles. The maximum i_{\max} is the double angle between the Earth's axes of rotation and orbit (N and S), i.e., 0.408 rad = 23.4° .

Then the angle φ_Ω increased on (Fig. 5) to reach 2π . The 2π value was subtracted from φ_Ω in the plots (at $\varphi_\Omega > 2\pi$), and the fall of i to zero corresponded to passage through 2π . Thus, the φ_Ω growth continued until $\varphi_\Omega \approx \pi/2 + 2\pi$ and then broke at $i = 0$. This behavior of φ_Ω indicates clockwise rotation of the orbit at T_{pr} , with almost six full rotations in 1000 year (Fig. 5).

During the interval of 1000 years the eccentricity reached $e = 0.0015$, with cycles ($T_{e2} = 10$ years) never longer than 10 years (Fig. 5).

The eccentricity e and the rotation cycle increased between 0 and 110 kyr at every 2 kyr while i_{\max} remained within 0.815 rad; the eccentricity likewise showed cyclic behavior and reached the maximum of $e = 0.0035$.

Models 2 and 3 were explored in the same way and likewise showed aperiodic change of φ_Ω in the beginning, eccentricity cycles, and orbit rotation. The results for model 3 were, however, slightly different: the first eccentricity cycle

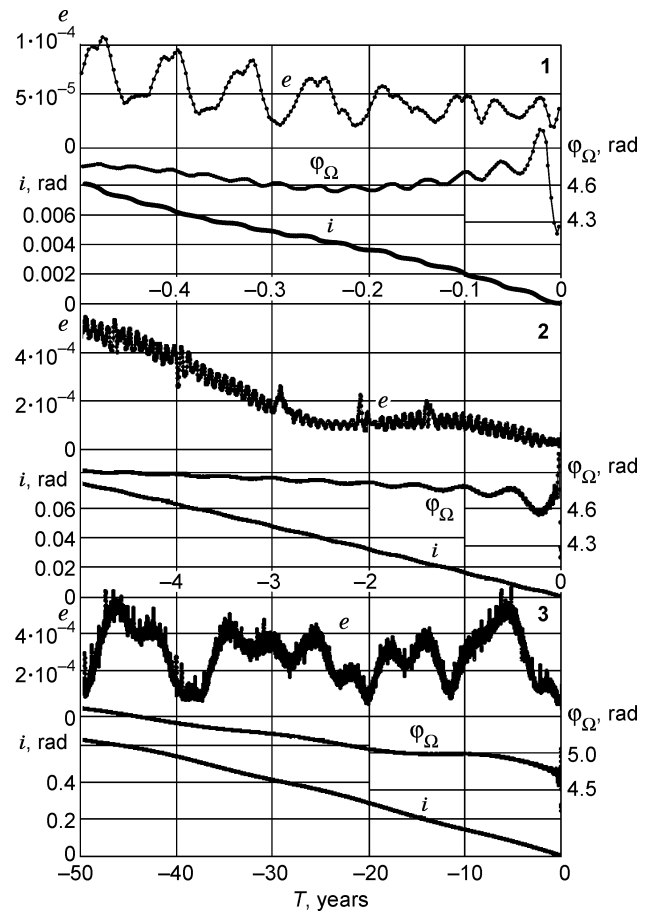


Fig. 4. Evolution of body orbital elements (e , φ_Ω , i) at each rotation in model 1 at time intervals: 1 — 0...–0.5 year; 2 — 0...–5 years; 3 — 0...–50 years. T are Julian dates since 30.12.1949; step size corresponds to orbital period.

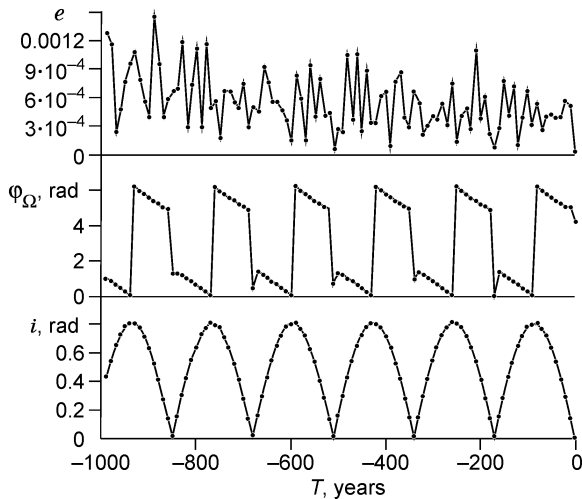


Fig. 5. Evolution of body orbital elements (e , φ_{Ω} , i) at each rotation in model 1. Time interval is 1000 years, step size is 10 years.

was $T_{e1} = 14.1$ days, and 143 year cycles set up since some time; the orbit rotation cycle was $T_{pr} = 2604$ years.

Orbital precession of a peripheral body. The orbital elements i and φ_{Ω} can be used to find the projections of the unit vector of the body's axis \mathbf{N} in the compound model onto the fixed equatorial coordinate axes xyz (Fig. 1):

$$N_x = \sin i \cos \varphi_{\Omega}, \quad N_y = -\sin i \sin \varphi_{\Omega}, \quad N_z = \cos i, \quad (12)$$

where $N = \sqrt{N_x^2 + N_y^2 + N_z^2} = 1$.

The dashed circle in Fig. 1 marks the orbital planes of both the Earth and the model bodies in order to simplify the image. Let φ_E be the position of the ascending node of the Earth's moving orbit in the xyz coordinates and i_E be the orbit inclination relative to the fixed equatorial plane. Let the axis z_E in the coordinates $x_{EY_E}z_{E}$ related to the moving Earth's orbit be directed along the Earth's orbit axis \mathbf{S} and the axis x_E cross its ascending node γ in the fixed equatorial plane (moving ecliptic coordinates). Then, the projections of the body's axis \mathbf{N} onto the axes $x_{EY_E}z_{E}$ (Fig. 1) are

$$N_{xE} = N_x \cos \varphi_E + N_y \sin \varphi_E, \quad (13)$$

$$N_{yE} = N_x \sin \varphi_E \cos i_E + N_y \cos \varphi_E \cos i_E + N_z \sin i_E, \quad (14)$$

$$N_{zE} = N_x \sin \varphi_E \sin i_E + N_y \cos \varphi_E \sin i_E + N_z \cos i_E. \quad (15)$$

The unit vector \mathbf{N} for model 1 plotted in the plane x_{EY_E} over two time intervals of 0...–1000 years and 0...–110 kyr rotates around \mathbf{S} (Fig. 6), and the data imaged relative to the fixed Earth's orbit are splitting. Testing the other models in the same way indicated that the Earth's spin axis \mathbf{N} in this compound model precesses about the axis \mathbf{S} . The precession cycles define the cycles of I and φ_{Ω} and are $T_{pr} = 170$ years for models 1 and 2 and $T_{pr} = 2604$ years for model 3.

Spin axis nutation in the Earth's compound model. The angle between the moving rotation axes \mathbf{N} and \mathbf{S} in the Earth's compound model is

$$\varepsilon = \arccos N_{zE}. \quad (16)$$

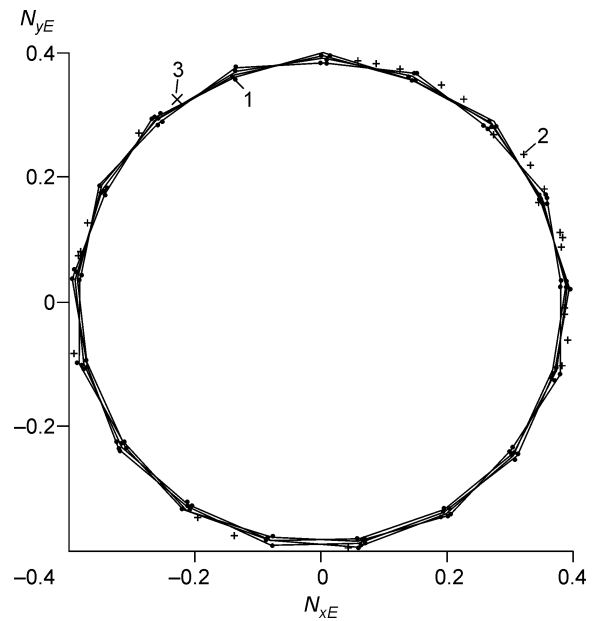


Fig. 6. Precession of unit vector \mathbf{N} of peripheral body (model 1) around moving orbital axis \mathbf{S} in different time intervals: 1 — 0...–1000 years; 2 — 0...–110 kyr; 3 — position of \mathbf{N} during end of interval 0...–110 kyr.

We calculated the angles ε and studied their evolution for all time intervals in the three models. For the shortest cycles we took the difference $\Delta\varepsilon = \varepsilon - \varepsilon_s$, where ε_s is the ε sliding average obtained by averaging over an interval equal to the double first cycle. The nutation cycles are shown for model 3 (4 in Fig. 7). In the 0...–1 year interval, the cycle is $T_{n1} = 13.9$ days, which must be due to the lunar effect because T_{n1} is half the Moon's orbiting cycle. The nutation in the 0...–4 interval must be due to the Sun its cycle $T_{n2} = 0.5$ year being half the Earth's orbiting cycle.

The period T_{n3} in the 0...–50 year interval corresponds to the rotation of lunar orbital nodes. This is the cycle at which the Moon's orbit oscillates with respect to the ecliptic plane reaching its known "low" and "high" positions. Nutation at $T_{n3} = 18.6$ years characterizes the major lunar pull on the Earth's spin axis.

The longest nutation cycle of $T_{n4} = 2580$ years appeared in the interval 0...–6 kyr, where the nutation amplitudes increased with period. Note that we additionally obtained nutation cycles of 220 years, or less than T_{n4} (4 in Fig. 7). However, those cycles were rather apparent than real: that was a longer-period image of the $T_{n3} = 18.6$ year cycles of the curve points sampled at every 20 years.

The maximum nutation cycles ε_1 over the 0...–40 kyr interval (scale T_1 , 4 in Fig. 7), with a time step of 1000 years, almost never exceeded the cycles ε . Therefore, there were no significant cycles larger than $T_{n4} = 2580$ years in this interval. The same modeling for the other cases (Table 1) showed cycles like T_{n1} to T_{n3} but no cycles of $T_{n4} = 2580$ years. We tested the 0...–110 kyr interval in model 1 but found no nutation with a significant amplitude and a period more than T_{n3} . More tests were applied to look into the 10 kyr end of the 0...–110 kyr interval where continuous rotations were at a

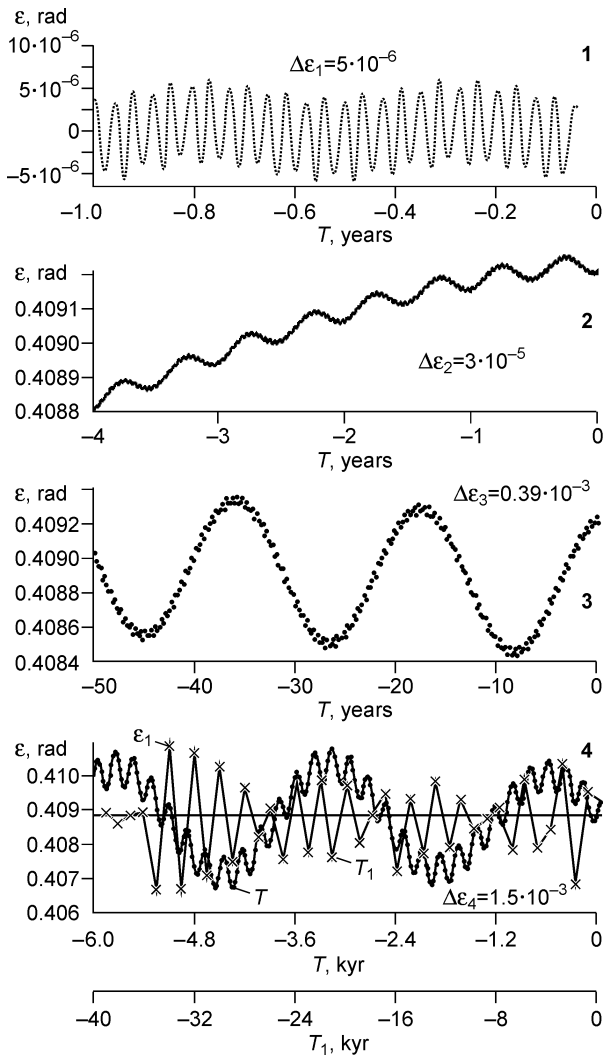


Fig. 7. Nutation cycles of Earth's rotation axis in different time intervals predicted by compound model. $\Delta\epsilon$ is deviation of nutation angle from sliding average. See text for explanation.

cycle of 28 years. The nutation cycles were the same (T_{n1} , T_{n2} , T_{n3}) as in Table 1. Thus, the ϵ nutation cycles were invariable throughout the 110 kyr interval.

Discussion

Checking against other results. We developed a new method to derive differential equations of the Earth's rotational

Table 1
Precession and nutation parameters in three models of Earth's rotation

Model	T_{pr} , years	T_{e1} , days	T_{n1} , days	$\Delta\epsilon_1 \cdot 10^{-6}$, rad	T_{n2} , years	$\Delta\epsilon_2 \cdot 10^{-5}$, rad	T_{n3} , years	$\Delta\epsilon_3 \cdot 10^{-3}$, rad	T_{n4} , years	$\Delta\epsilon_4 \cdot 10^{-3}$, rad
1	170	26	13.9	75	0.5	50	19.8	7.8	—	—
2	170	26	13.9	75	0.5	50	19.8	7.8	—	—
3	2604	27	13.9	5	0.5	3	18.6	0.39	2580	1.5
DERM	—	—	—	—	0.5	0.268	18.6	0.045	—	—

Note. $\Delta\epsilon_i$ is nutation amplitude; DERM — according to solutions of differential equations of rotational motion.

motion and integrated them numerically to investigate the effect of the Sun and the planets on the Earth (Smul'skii and Sechenov, 2006). We found out (Smul'skii and Sechenov, 2007) that the cyclic effect of the celestial bodies on the Earth's rotation axis was due to the periodicity at which they cross the equatorial plane, i.e., their orbiting half-periods. This inference is perfectly consistent with the nutation cycles T_{n1} and T_{n2} related to the passage of the Moon and the Sun through the Earth's equator as obtained for the compound model.

Note also a particular point concerning the reported results. The neighbor $\Delta\epsilon$ oscillations (1 in Fig. 7) are not identical but repeat at T_{n1} . Thus, there may be another period $2T_{n1}$ to modulate the $\Delta\epsilon$ cycles. The same is the case of T_{n2} cycles (2 in Fig. 7) if we represent them as the difference $\Delta\epsilon$. These modulations at periods $2T_{n1} = 1$ month and $2T_{n2} = 1$ year are due to the ellipticity of the lunar and solar orbits with respect to the Earth. As a result, the two bodies cause different effects on the Earth's rotation when they pass through different sides of the equatorial plane. These double cycles are commonly used in processing observatory data.

For comparison, Table 1 lists major nutation periods and amplitudes reported by Bretagnon et al. (1997) as complete Earth's rotation solutions for a short time interval. Our solution for the case of the solar effect only gives a similar cycle of $T_{n1} = 0.5$ year and an amplitude of $\Delta\epsilon_2 = 0.27 \cdot 10^{-5}$ rad. Thus, the nutation cycles T_{n2} and T_{n3} (Table 1) derived from the compound Earth rotation model coincide with those obtained by solving the differential equations of rotational motion. The amplitudes depend on model parameters and decrease proportionally to the orbital radius a ; for the Earth they are an order of magnitude smaller than in model 3.

As we wrote above, the theories of orbital climate forcing invoke nutation cycles of $T_\epsilon = 41.1$ kyr, with amplitudes $\Delta\epsilon = 1.7 \cdot 10^{-2}$ rad = 0.98° for the first ~400 kyr. The compound Earth's rotation models, however, predict much smaller amplitudes and shorter periods (Table 1).

Thus, the nutation cycles are the same in the compound models and in the solutions of second-order differential equations of the Earth's rotation, and are conformable with observation. Yet, the compound models shows no $T_\epsilon = 41.1$ kyr cycles which were predicted by solving simplified first-order differential equations of rotational motion.

Possible mechanism of continent interaction. We studied the evolution of the Earth's axis using a model in which a part of the Earth's mass is divided between bodies distributed

uniformly in its equatorial plane. Thus, it is, in a sense, a model of “a dispersed Earth”. Note that a similar model can be found in Isaac Newton’s works. As Laplace (1984) wrote, Newton considered the Earth as a spheroid protuberant at the equator, which is composed of a sphere and a meniscus around it. The meniscus consists of orbiting particles, and the nodes of their orbits are supposed to undergo retrograde motion arising from the combined action of the Sun and the Moon. From the connection of those bodies together there should succeed a retrograde motion of the whole meniscus at the intersection of the equator and the ecliptic, called *the precession of the equinoxes*; the retrograde motion is slow because the meniscus divides it with the sphere to which it is attached. Nobody before Newton could ever suspect the reason of the great phenomenon depending on the compression of the Earth and on the retrograde motion of meniscus nodes caused by the Sun. These were two effects Newton was the first to discover (Laplace, 1984). Thus, Newton likewise “decomposed” the Earth into separate bodies.

This model of Earth rotation allowed us to predict nutation oscillations in addition to the precession of the Earth’s axis, as well as cyclic changes of the distance between the peripheral bodies. In model 3 the bodies behave as rhythmic convergence and divergence (Fig. 3), which holds also in models 1 and 2. Hence, convergence and divergence of bodies is a general property of a rotating system. One can expect the same motion for bodies lying on the Earth’s surface, which actually corresponds to the case of model 3 with $a = R_{Ee}$.

Everybody knows that the eastern borders of North and South America mimic the contours of western Europe and Africa. This striking match has been explained in various hypotheses, and our modeling drives us to another explanation. We think whether the multiple convergence cycles could shape up the contours of the continents to adjust them to each other. These motions should occur in the plane orthogonal to the Earth’s rotation axis, i.e., in the way the bodies move in the compound model.

The continents, treated as bodies rising above the Earth, should experience the gravity pull from the Moon, the Sun, and the planets, according to the prediction of our model. However, unlike the peripheral bodies in our model, the continents are not free but rooted deeply in the Earth’s interior, so that their motion has to overcome viscous friction. As a result, the convergence-divergence cycles can grow as long as geological periods.

The bodies in the compound model approach each other as close as one third of their orbit size. They never collide being much smaller than the continents, which are subject to collision. Thus, our model has implications for continent interaction and may explain why the continent contours match so strikingly.

Conclusions

We have justified and formulated a compound model of Earth rotation and explored three models of a rotating Earth

for (i) the time corresponding to 20,000 continuous rotations of a peripheral body and (ii) rotations at equal cycles within a time span of 110 kyr.

The orbital axes precess (rotate against the orbital motion) around the moving Earth’s orbit at cycles T_{pr} from 170 to 2604 years in different models.

The model Earth’s spin axis oscillates at periods of 14 days, 0.5 year, and 18.6 years in all models and also at 2508 years in model 3. The first three nutation cycles are consistent with solutions of the rotation equations and with observed data.

The three models predict no 41.1 kyr obliquity cycle invoked in the existing theories of orbital climate forcing.

In their orbital motion, the peripheral bodies in the compound model converge and diverge cyclically. Applied to the case of continent interaction, these convergence-divergence cycles and the ensuing periodic collisions may explain the perfect fit of continental margin geometries between two Americas on the one side and Europe and Africa on the other side.

All labor-consuming computation was run on an MCS1000 supercomputer at the Siberian Supercomputer Center, Siberian Branch of the Russian Academy of Sciences (Novosibirsk), where people were very helpful.

The paper profited much from constructive criticism and editorial efforts of E.A. Grebenikov from the Computer Center of the Russian Academy of Sciences (Moscow).

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Editorial responsibility: A.E. Kontorovich