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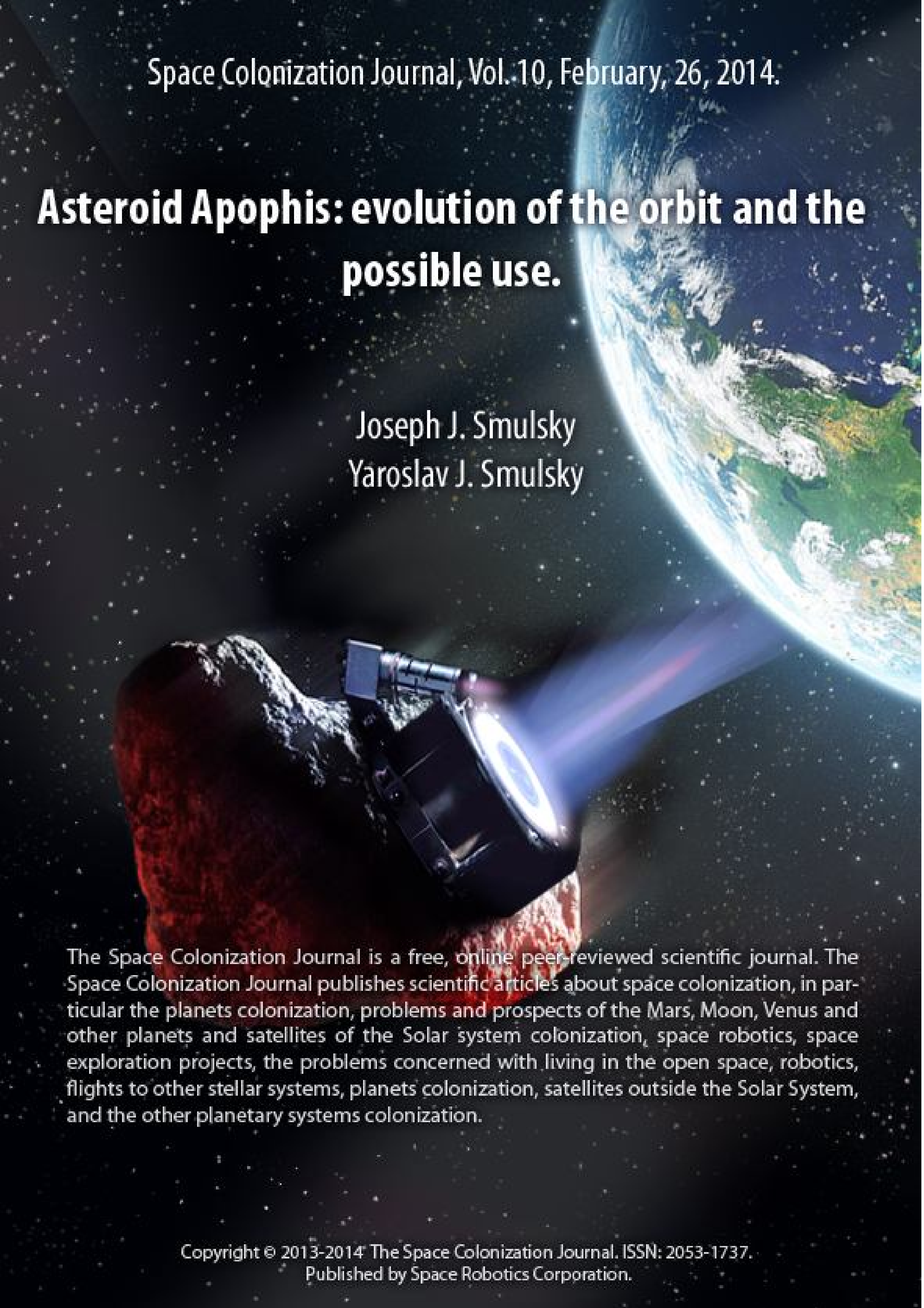

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Asteroid Apophis: evolution of the orbit and the possible use.

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Abstract. Based on the analysis of available scientific datasets, the author found that a number of uncertainties in the trajectory of Apophis is caused by imperfections of methods for its determination. A New numerical method was used to integrate differential equations of motion of Apophis, planets, the moon and the sun and the evolution of the asteroid's orbit was studied. In April 13, 2029 Apophis will pass by the Earth at a distance of 6 of its radius and within 1,000 years it will not come closer. Conditions to transform Apophis into an Earth satellite are calculated; it can be used for a variety of space exploration tasks.

Keywords: asteroid, Apophis, motion, orbit, satellite, space, space, Earth.

1. Introduction.

In [1] the author describes the history of the problem. For the first time the Apophis asteroid was discovered June 19-20, 2004 at the Kitt Peak Observatory [2] and was re- observed in Siding Spring Survey Observatory [3] On December 20 the same year. Since then, it attracted the attention of many researchers. According to the results of the first determinations of the elements of its orbit, calculations of its motion were performed, and it was found [4] that on April 13, 2029 it will be at a distance of 38,000 km from the center of the Earth. As a result of the Earth's influence, the orbit of the asteroid will change significantly. Since existing methods of calculation give large errors, then, according to some researchers [1,4,5], its trajectory becomes uncertain, indeterminate and even chaotic. Various static analysis methods predict some probability of collision of Apophis with Earth on April 13, 2036. The main interest of researchers has focused onthis problem.

In [4] the author analyzes the possibility of Apophis's approach to the Earth and its consequences. Various countermeasures against asteroid's fall on Earth are examined and reconnaissance missions are proposed. In conclusion, it is noted that it's needed to forecast the Apophis motion for 2029 with maximum errors that lie within one kilometer.

Issues related to observation of Apophis with terrestrial and space systems, are discussed in several papers [1, 4, 5, 6]. Since the orbit of the asteroid is near to the Earth's orbit, in a large part of the orbit the asteroid disk is illuminated partially or not at all visible. Therefore, it's necessary to establish periods of ground-based observations, and for use of space means it is required to determine the most effective location of them in the orbit. Calculation of asteroid's motion is apparently one of the most difficult problems. In [7] differential equations of perturbed motion of the asteroid are integrated by the Everhardt method [8], and the positions of the perturbing bodies were taken from ephemeris DE403 and DE405 JPL USA. The authors paid attention to resonance phenomena, which can lead to a collision of Apophis with the Earth in 2036. The authors of [9, 10] based on 933 observations have improved the initial orbital parameters of Apofis. However, in the study using standard methods they found that because of the passage of the asteroid through a series of resonances with Earth and Mars its motion becomes chaotic.

In order to determine the likelihood of Apophis collision with Earth in 2036 they made 10 thousand variations of the initial conditions, 13 of which lead to a collision of Apophis with Earth [10].

E. A. Smirnov [11] aimed at testing different methods of integration for their suitability to calculate the motion of the asteroid that could collide with the Earth. He considered the following methods: Everhart, Runge-Kutta of 4th order, Yoshida of 6 and 8 orders, Hermite of 4 and 6 orders, Multistep Predictor-Corrector (MS-PC) method of 6 and 8 orders and method of Parker-Sochacki. The author concludes that the Everhardt method and the MS-PC method are inferior to other methods. He also notes that in these problems with singular points finite difference methods poorly approximate higher derivatives. This finding is very important, as further there will be the results of integrating the equations with another method that does not have these disadvantages. Note that EA Smirnov [11] replaced the Everhardt method with the Runge-Kutta method when Apophis was approaching the Earth . Since the first method is widely used in the integration of differential equations of motion of bodies in the solar system, it is necessary to pay attention to its mistakes in the problems of convergence of bodies.

Paper [12] examines mathematical problems of calculating the orbit of the asteroid and its change with different methods. The possibilities of shock and thermo-kinetic methods of correcting the trajectory of Apophis were assessed. Extensive research of Apophis are presented in [1]. All the observational history is given chronologically and dynamics of reduction of errors in its orbital elements is given. The authors paid close attention to the accuracy of the orbit calculation and the influence of different factors. The effect of inaccurate position of planets, physical parameters of the asteroid, perturbations of other asteroids. The influence on the accuracy of the integration of number length, the non-sphericity of the Earth and the Moon, the perturbations from solar radiation and uneven heating, etc. were examined. Equations of the perturbed motion of the asteroid were integrated using the standard dynamic model (SDM), in which positions of other bodies were taken from ephemeris DE405. It is known that the DE405 ephemeris are approximation of several hundreds of thousands of observations until 1998. When switching to the ephemeris DE414, which approximate the observation data up to 2006, the prediction error of the trajectory of Apophis up to 2036 has decreased by 140 thousand km. This error, as shown in [1], is ten times larger than errors from other disturbances. Note that this result demonstrates the need for a more accurate way to calculate the trajectory of the asteroid.

Paper [1] in great detail discusses further research to clarify the trajectory of Apophis. Optical and radar measurements are scheduled for each year, observation programs are planned for Apophis approaches in 2021 and 2029 and spacecraft missions in 2018 and 2027 are planned. Reduction values for asteroid's trajectory errors that would result from the previously mentioned actions are assessed. It should be noted that the ephemeris built on approximation of observations data allow to detect with good precision the position of bodies within the approximation interval. The farther it is from the approximation interval, the more deteriorated bodies' positions prediction is at the time removed from this interval. . Therefore observations and missions to Apophis planned in [1] will be used to improve future ephemeris. So, when calculating the trajectory of Apophis integrated equations of perturbed motion [1,7,12] were used, and positions of other bodies were taken from ephemeris. We used difference methods of integration that produce large error in determining the higher derivatives when bodies approach each other. Adding other weak interactions to the basic Newtonian gravitational effects leads to complication of the problem and increases the uncertainty of the trajectory. Many weak effects do not have enough quantitative reasoning. In addition, physical parameters of the asteroid and the coupling constant are unknown. Therefore, expert surveys are used when accounting for these effects. And the most significant is that the error in the solution of the problem of the asteroid motion in case of the Newtonian interaction is by orders greater than additive effects from small extra effects.

To study the effect of initial conditions on the probability of Apophis collision with Earth, methods of [1] are used. The initial conditions for the asteroid are determined from the elements of its orbit, which are known with some error σ. For example, the amount of eccentricity $e=e_n \pm \sigma_e$, where e_n – nominal value of the eccentricity, σ_e – root-mean-square deviation in the processing of several hundreds of observations of the asteroid. In these works, collision parameters are searched for in the area of possible movements of the asteroid 3σ, for example, initial conditions for eccentricity are calculated in the area $e=e_n\pm 3\sigma_e$.

10 thousand, and in some studies 100 thousand sets of initial conditions are randomly selected from it , i.e. 10 or 100 thousand asteroids are considered instead of a single asteroid. Some of them may collide with the Earth. The probability of asteroid collision with Earth is determined based on their number. Such a statistical statement is incorrect. If there are many measurements of a parameter, its nominal value, e.g., e_n is the most probable. Therefore, the trajectory calculated using the nominal initial conditions (IC) is the most probable. Trajectory calculated with a small deviation from the nominal IC, is less probable, and the probability of the trajectory calculated using parameters of the boundary of the deviations area (i. e. $e=e_n\pm\sigma_e$) tends to zero. The trajectory with IC defined by parameters that are three times more than their possible deviations (i.e. per $e=e_n\pm 3\sigma_e$) has an even smaller probability. Since IC are determined by 6th orbital elements, the simultaneous realization of the boundary values by all elements is even less probable. In our view, the influence of IC should be examined on those sets that are prepared by successive accumulation of observation data. If the difference between the movements of the asteroid on the last two sets of IC is insignificant until a certain date, it can be concluded that up to this date the motion of the asteroid is completely determined by the initial conditions.

As it's shown in [1], to further clarify the trajectory of Apophis it's needed to perform some additional work. It is therefore interesting to define its trajectory more precisely, which will reduce the number of trajectories.

For integration of differential equations of motion of bodies in the solar system for large time intervals [13-14], we have developed the Galactica program. It takes into account only the force of gravity, and differences are not used to determine derivatives. In problems of the composite model of the Earth's rotation [15] and gravitational maneuver near Venus [16] equations of motion with a small distance (of the order of the radius of the planet) between the bodies were integrated. As a result of these tasks and multiple tests is has been found that using Galactica it's possible to calculate the motion of Apophis before its approach to the Earth, and after it with a sufficiently high accuracy. Therefore, in the present work we have studied the evolution of the Apophis orbit, in the result of which some prospects for its use have been revealed.

2. Statement of the problem.

In the interaction of the asteroid, the Sun, Moon and planets according to the law of gravitation, differential equations of motion are as follows [17]:

$$\frac{d^2\vec{r}_i}{dt^2} = -G\sum_{k\neq i}^{n}\frac{m_k\vec{r}_{ik}}{r_{ik}^3}, \quad i=1,2,\ldots,n, \tag{1}$$

where $\vec{r}_i$ - the radius vector of a body with mass m_i in relation to a non-accelerated center, for example, the barycenter of the Solar System; G – gravitational constant; $\vec{r}_{ik} = \vec{r}_i - \vec{r}_k$ – radius vector to a body with mass m_i from body with mass m_k. n = 12 (nine planets, the sun, the moon and the asteroid). As a result of numerical experiments and analysis, we concluded that the finite-difference methods of integration do not provide the necessary accuracy. The algorithm of the program Galactica calculates the function value of the next point of time $t=t_0+\Delta t$ using a Taylor series, which, for example, has the following form for the x-coordinate:

$$x = x_0 + \sum_{k=1}^{K}\frac{1}{k!}x_0^{(k)}(\Delta t)^k, \tag{2}$$

where $x_0^{(k)}$ – derivative of order k at time t_0. Speed x' is determined by a similar formula, and acceleration x_0'' by formula (1). Higher derivatives $x_0^{(k)}$ are determined analytically by differentiating equations (1). Now we use the calculation scheme of the sixth order, i.e. K=6.

3. Preparation of the initial conditions.

The problem is considered in a barycentric equatorial coordinate system for the epoch J2000.0 the Julian day $JD_s = 2451545$. Apophis orbital elements (eccentricity e, semimajor axis a, angle to the plane of the ecliptic i_e, angle of the ascending node Ω, argument of perihelion ω_e, etc.) and its positions (mean anomaly M) were taken from the base of small planets bodies [18] 30.0 in November, 2008. They are presented by 16 decimal signs and are listed in table 1 as three options. Now, the first option is being considered. These elements correspond to the solution with the number JPL sol. 140, which is obtained by Otto Mattic on 04.04.2008. Table 1 also gives uncertainty σ of these data. Their relative value δ is within the range $2.4\cdot10^{-8} \div 8\cdot10^{-7}$. These data are available in the database of asteroids by Edward Bouella [19], but they are represented by 8 decimal signs and differ from previous elements in the seventh sign, i.e. within the error δ. Paper [1] used the orbital elements of Apophis for epoch JD = 2453979.5 (01.0 in September 2006), which correspond to the solution with the number JPL sol. 142. According to the Horizons system publicly available to JPL, the solution sol. 142 can be extended to 30.0 in November 2008. In this case, it is clear that the difference between the orbital elements of solution 142 and solution 140 doesn't exceed $0.5\cdot\sigma$ *of* uncertainties of the orbital elements. Using elements shown in table 1, Cartesian coordinates and velocities of Aphophis in a barycentric equatorial system were calculated per the following algorithm [14, 15, 20, 21]. From Kepler's equation:

$$E - e\cdot\sin E = M \tag{3}$$

the eccentric anomaly E is determined, and the true anomaly φ_0 is calculated on its basis

$$\varphi_0 = 2\cdot\mathrm{arctg}[\sqrt{(1+e)/(1-e)}\cdot\mathrm{tg}(0.5\cdot E)], \tag{4}$$

In further calculations we used the results of interaction between two bodies (the Sun and the asteroid) [16, 21]. The body trajectory equation in a polar coordinate system with the origin in the Sun, is of the following form:

$$r = \frac{R_p}{(\alpha_1+1)\cos\varphi - \alpha_1}, \tag{5}$$

where the polar angle φ (in astronomy it's the true anomaly) is measured from the perihelion $r = R_p$; $\alpha_1 = -1/(1+e)$ - trajectory parameter; $R_p = a\cdot(2\alpha_1+1)/\alpha_1$ – radius of perihelion.

Table. 1. Three variants of the orbital elements of Apophis [18] into two epochs in the heliocentric ecliptic coordinate system, 2000 since $JD_S = 2451545$.

Element	First option, November, 30, 2008. $JD_{01} = 2454800.5$ JPL sol.140	Second option, January 4, 2010. $JD_{02} = 2455200.5$ JPL sol.144	Third option November 30, 2008. $JD_{01} = 2454800.5$ JPL sol.144	Uncertainty $\pm\sigma$ 1st Var.	Units
	Значение				
e	.1912119299890948	.1912110604804485	.1912119566344382	7.6088e-08	
a	.9224221637574083	.9224192977379344	.9224221602386669	2.3583e-08	a.u.
q	.7460440415606373	.7460425256098334	.7460440141364661	8.6487e-08	a.u.
i_e	3.331425002325445	3.331517779979046	3.331430909298658	2.024e-06	degree
Ω	204.4451349657969	204.4393039605681	204.4453098275707	0.00010721	degree
ω_e	126.4064496795719	126.4244705298442	126.4062862564680	0.00010632	degree
M	254.9635275775066	339.9486156711335	254.9635223452623	5.7035e-05	degree
t_p	2454894.912750123770 (2009-Mar-04.41275013)	2455218.523239657948 (2010-Jan-22.02323966)	2454894.912754286546 (2009-Mar-04.41275429)	5.4824e-05	JD day
P	323.5884570441701 0.89	323.5869489330219 0.89	323.5884551925927 0.89	1.2409e-05 3.397e-08	day year
n	1.112524233059586	1.112529418096263	1.112524239425464	4.2665e-08	degree/day
Q	1.098800285954179	1.098796069866035	1.098800306340868	2.8092e-08	a.u.

Expressions for the radial velocity υ_r and transversal velocity υ_t have the form:

$$\upsilon_r = \upsilon_p \sqrt{(\alpha_1 + 1)^2 - (\alpha_1 + 1/\bar{r})^2} \text{, where } \varphi > \pi \;\; \upsilon_r < 0; \qquad \upsilon_t = \upsilon_p / \bar{r}, \tag{6}$$

where $\bar{r} = r/R_p$ – relative radius, and the velocity at perihelion is

$$\upsilon_p = \sqrt{G(m_S + m_{As})/(-\alpha_1 R_p)}, \tag{7}$$

where $m_S = m_{11}$ – mass of the Sun (m_{11} see table. 2), and $m_{As} = m_{12}$ – mass of the asteroid.

Body travelling time in an elliptical orbit, from the point of perihelion to the position in orbit with a radius $\bar{r}$, is determined by the formula:

$$t = \frac{R_p}{v_p}\left[\frac{\bar{r}\,|\bar{\upsilon}_r|}{2\alpha_1 + 1} - \frac{\alpha_1\left(\pi/2 + \arcsin\{[(2\alpha_1 + 1)\bar{r} - \alpha_1]/(-\alpha_1 - 1)\}\right)}{(-2\alpha_1 - 1)^{3/2}}\right], \tag{8}$$

where $\bar{v}_r = v_r / v_p$ – relative radial velocity.

At the initial time $t_0 = 0$, which corresponds to the epoch JD_0 (see table. 1), the polar radius r_0 of the asteroid is calculated by the formula (5) depending on the initial polar angle (true anomaly) φ_0. The initial radial and transversal velocities are determined by the formulas (6), depending on r_0. Cartesian coordinates and velocities in the asteroid orbit plane (axis x_o passes through perihelion) is calculated by the formulas:

$$x_o = r_0 \cdot \cos\varphi_0; \;\; y_o = r_0 \cdot \sin\varphi_0; \tag{9}$$

$$\upsilon_{xo} = \upsilon_r \cdot \cos\varphi_0 - \upsilon_t \cdot \sin\varphi_0; \;\; \upsilon_{yo} = \upsilon_r \cdot \sin\varphi_0 + \upsilon_t \cdot \cos\varphi_0. \tag{10}$$

Asteroid Coordinates in a heliocentric ecliptic coordinate system are defined as follows:

$$x_e = x_o \cdot (\cos\omega_e \cdot \cos\Omega - \sin\omega_e \cdot \sin\Omega \cdot \cos i_e) - y_o \cdot (\sin\omega_e \cdot \cos\Omega + \cos\omega_e \cdot \sin\Omega \cdot \cos i_e); \tag{11}$$

$$y_e = x_o \cdot (\cos\omega_e \cdot \sin\Omega - \sin\omega_e \cdot \cos\Omega \cdot \cos i_e) - y_o \cdot (\sin\omega_e \cdot \sin\Omega - \cos\omega_e \cdot \cos\Omega \cdot \cos i_e); \tag{12}$$

$$z_e = x_o \sin\omega_e \cdot \sin i_e + y_o \cdot \cos\omega_e \cdot \sin i_e. \tag{13}$$

Components of the asteroid velocity $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} in this coordinate system are calculated using formulas similar to (11) - (13). Since equations (1) are treated in the equatorial fixed coordinate system, then the ecliptic coordinates (11) - (13) are converted to equatorial by formulas:

$$x = x_e; \;\; y = y_e \cdot \cos\varepsilon_0 \cdot - z_e \cdot \sin\varepsilon_0; \; z = y_e \cdot \sin\varepsilon_0 + z_e \cdot \sin\varepsilon_0, \tag{14}$$

where $\varepsilon_0 \cdot$ - inclination between the ecliptic and the equator in the epoch *JDs*. Velocity components $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} are transformed into the equatorial coordinate system by the formulas similar to (14).

In these calculations, 6 orbital elements from table 1 are used, namely: e, a i_e, Ω, ω_e and M. Others were used for control. The radius of perihelion R_p and aphelion $R_a = -R_p/(2\alpha_1+1)$ were compared with q and Q, respectively. Orbital periods were calculated by the formula (8) as double travelling time from perihelion to aphelion at $r = R_a$. The same equation was used to calculate the time of perihelion passage at $r = r_0$. These two parameters were compared with P and t_p from table 1, respectively. The greatest relative difference in Q and q did not exceed $1.9 \cdot 10^{-16}$, and in P and t_p it was less than $8 \cdot 10^{-9}$. Positions and velocities of the planets and the moon on JD_0 epoch are defined by *JPL*-theory DE406/LE406 [22-23]. Masses of these bodies have been modified in [14], and the mass of the asteroid is calculated as for a ball with a diameter d=270 m and density ρ=3000 kg/meter^3. Masses of all the bodies and the initial conditions are shown in table 2. Note that the whole algorithm (3)-(14) for preparing and checking the initial conditions is implemented in the AstCoor2.mcd program in MathCad.

Table. 2. Planet Masses m_{bj} from Mercury to Pluto, the Moon, the Sun, and Apophis and the initial conditions for the epoch JD_0 = 2454800.5 (November, 30, 2008) in the heliocentric equatorial coordinate system for the epoch 2000.0, JD_S = 2451545. G = 6.67259E-11 meter^3/(sec^2·kg).

Bodies j	Body weights in kg per m of the coordinates and velocities in metr/s			
	m_{bj}	x_j, v_{xj},	y_j, v_{yj}	z_j, v_{zj}
1	3.30187842779737E+23	-17405931955.9539	-60363374194.7243	-30439758390.4783
		37391.7107852059	-7234.98671125365	-7741.83625612424
2	4.86855338156022E+24	108403264168.357	-2376790191.8979	-7929035215.64079
		1566.99276862423	31791.7241663148	14204.3084779893
3	5.97369899544255E+24	55202505242.89	125531983622.895	54422116239.8628
		-28122.5041342966	10123.4145376039	4387.99294255716
4	6.4185444055007E+23	-73610014623.8562	-193252991786.298	-86651102485.4373
		23801.7499674501	-5108.24106287744	-2985.97021694235
5	1.89900429500553E+27	377656482631.376	-609966433011.489	-270644689692.231
		11218.8059775149	6590.8440254003	2551.89467211952
6	5.68604198798257E+26	-1350347198932.98	317157114908.705	189132963561.519
		-3037.18405985381	-8681.05223681593	-3454.56564456648
7	8.68410787490547E+25	2972478173505.71	-397521136876.741	-216133653111.407
		979.784896813787	5886.28982058747	2564.10192504801
8	1.02456980223201E+26	3605461581823.41	-2448747002812.46	-1092050644334.28
		3217.00932811768	4100.99137103454	1598.60907148943
9	1.65085753263927E+22	53511484421.7929	-4502082550790.57	-1421068197167.72
		5543.83894965145	-290.586427181992	-1757.70127979299
10	7.34767263035645E+22	55223150629.6233	125168933272.726	54240546975.7587
		-27156.1163326908	10140.7572420768	4468.97456956941
11	1.98891948976803E+30	0	0	0
		0	0	0
12	30917984100.3039	-133726467471.667	-60670683449.3631	-26002486763.62
		16908.9331065445	-21759.6060221801	-7660.90393288287

4. Study asteroid approaches to planets and the Moon.

The Galactica program provides an opportunity to determine the closest approach R_{min} of an asteroid to a celestial body at a predetermined interval ΔT. These studies were performed by integration of equations (1) with initial conditions given in table 2. The integration was performed on a supercomputer NKS-160 in Computing Center of the Siberian Branch of the Russian Academy of Sciences in Novosibirsk. At the same time, the Galactica program used an extended number length (34 decimal digits) and counting step $dT=10^{-5}$ years. Studies have been performed on three time intervals: 0÷100 years (Fig. 1, a), 0÷100years (Fig. 1, b) and 0÷1000 years (Fig. 1c).

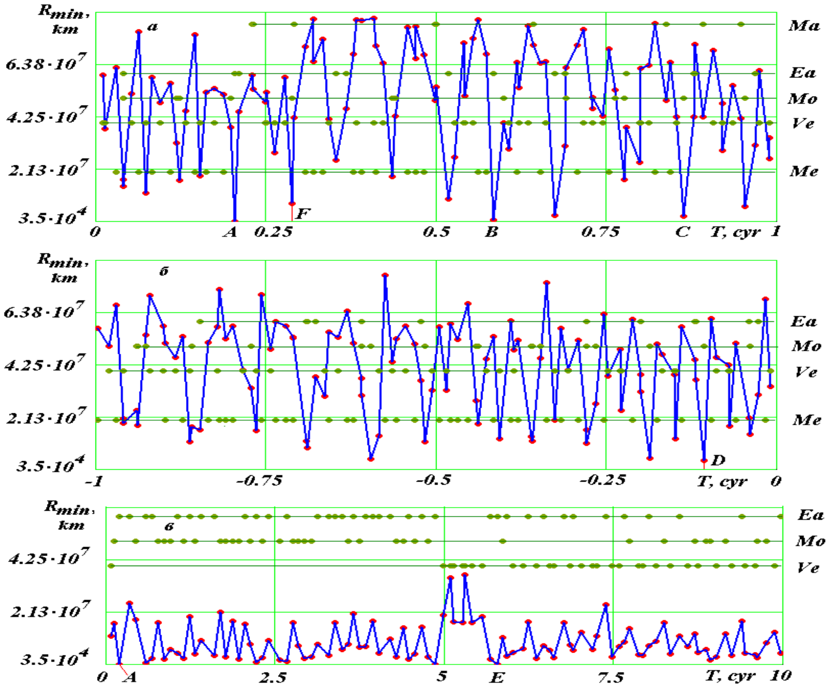


Fig. 1. Apophis approach for the time ΔT to the minimum distance (km) Rmin from celestial bodies: Mars (Ma), Earth (Ea), Moon (Mo), Venus (Ve) and Mercury (Me); a),b)- ΔT=1 year; c)ΔT=10; T, cyr is time in Julian centuries from epoch JD_0 (November, 30, 2008).

In the graphs of Fig. 1 points connected with a bold line show minimum distances R_{min} for approach of an asteroid to bodies, which are marked by dots connected with a horizontal line. That is, the point on the line is the minimum distance at which for time $\Delta T=1$ year an asteroid passed by the body that is indicated by point on the horizontal line at the same time. Fig. 1a shows that from November, 30, 2008 for 100 years there will be only one substantial convergence of Apophis with the Earth (point A) at T_A=0.203693547133403 centuries at a distance R_{minA}=38907 km. Next convergence (point B) is also with the Earth, but at T_B=0.583679164042455 centuries at a distance R_{minB}=622231 km, which is 16 times greater than the distance in the first approach. For other bodies, the closest approach is only with the Moon (point D) (see Fig. 1, b) at T_D=-0.106280550824626 centuries at a distance of R_{minD}=3,545,163 km.

Graphs in Fig. 1a and Fig. 1b show the minimum approaches of an asteroid to bodies at intervals ΔT=1 year. In integrating equations (1) in the time interval 1000 years (see Fig. 1c), minimum approaches of an asteroid to bodies were examined in the time interval ΔT=10 years. At these time intervals, approaches to Mercury and Mars did not appear, as at 10-year intervals the asteroid comes closer to other bodies. Also as in Fig. 1, there is an approach at T_A to the Earth. The second largest approach also occurs with the Earth in point E when T_E=5.778503 centuries at a distance R_{minE}=74002.9 km. During this approach the asteroid passes by the Earth at a distance that is almost twice larger than that at the time T_A.

In order to verify the results, equations (1) were integrated for 100 years with a double number length (17 decimal places) with the same step and with the extended number length with step $dT = 10^{-6}$ years. Integration accuracy (see table 3) is determined [14] by the relative change δM_z, where z is the projection of the angular momentum of the entire solar system for 100 years. As can be seen from the table, δM_z varies from $-4.5\cdot10^{-14}\div1.47\cdot10^{-26}$, i.e. 12 orders of magnitude. The last two columns of table 3 show the time differences between times of approach of the asteroid to the Earth in point A (see Fig. 1a) and the difference in distances with respect to the solution 1. In solution 2, with a shorter number length, approach time does not change, and a minimum distance has decreased by 2.7 meters . In solution 3, with the integration step reduced 10 times, the moment of convergence has changed by $-2\cdot10^{-6}$ years = -1.052'. Since this change is less than the step $dT = 1\cdot10^{-5}$ of solution 1 and is equal to two steps of solution 3, it is a clarification of the convergence time. The approach distance is also clarified by -1,487 km. According to updated estimates, Apophis's approach to the Earth occurs in 21 hours 44 minutes 45 seconds at a distance of 38,905 km. It should be noted that the results presented graphically in Fig. 1a for solutions 1 and 3 coincide completely. Small differences of solution 2 from solutions 1 and 3 are at T> 0.87 centuries.

Table 3. Comparison of the results of Apophis's approach to the Earth at different integration accuracies: L_{nb} – number length in decimal places.

Solution number	L_{nb}	dT, year	δM_z	T_{Ai}-T_{A1}, year	R_{minAi}-R_{minA1}, km
1	34	$1\cdot10^{-5}$	$1.47\cdot10^{-21}$	0	0
2	17	$1\cdot10^{-5}$	$-4.5\cdot10^{-14}$	0	$-2.7\cdot10^{-3}$
3	34	$1\cdot10^{-6}$	$1.47\cdot10^{-26}$	$-2\cdot10^{-6}$	-1.487

When integrating over the 1,000 years interval, the relative change in angular momentum is $\delta M_z=1.45\cdot10^{-20}$. As seen from solution 1 of table 3, this value exceeds δM_z 10 times in case of integrating at the interval of 100 years, i.e. error in case of an extended number length is proportional to time. This allows to estimate the error of the second approach of Apophis to Earth T_E=578 per results of calculations in the interval of 100 years the solution with steps $dT=1\cdot10^{-5}$ years and $1\cdot10^{-6}$ years. 88 years after the integration begins, the relative difference of the distances between the Earth and Apophis is $\delta R_{88} = 1\cdot10^{-4}$, which leads to errors in distance of 48.7 km in T_E =578 years. So, over a thousand year period of time Apophis will approach significantly only to the Earth. This will happen at time T_A of era JD_0. The approach time corresponds to Julian Day JD_A=2462240.406075 and calendar date April 13, 2029 at 21h 45' of the Greenwich Mean Time. The asteroid will pass within 38905 km from the center of the Earth, i.e. at a distance ~ 6 Earth radii. Next approach of Apophis to the Earth will happen in 578 years of age JD_0, when the asteroid will pass by the Earth at a distance of almost two times more than in the first approach. Calculated moment of approach of Apophis to Earth on April 13, 2029 coincides with the moments obtained in other works. For example, in a recent work [1] it is given to the nearest minute: 21h 45' UTC. The geocentric distance of passage is given in the range from 5.62 to 6.3 Earth radii, i.e. the distance of 6 Earth radii obtained by us is in this range. The coincidence of the results of calculations performed by various methods, on the one hand, indicates the validity of the event. On the other hand, these calculations are performed with different initial orbital elements of Apophis (as noted earlier, they differ in the 4-5 digit), so further refinement of the orbital elements, apparently, will not substantially change the results of approach in 2029.

As for the approach of Apophis to the Earth in 2036, as seen from Fig. 1a, it will not happen. An approach of Apophis close in time to point F at a distance of 7.26 million km to the Moon will happen September 5, 2037.

5. Evolution of the Apophis orbit.

When integrating the equations of motion (1) on the interval –1 century ≤ T ≤ 1 century, coordinates and velocities of the bodies over each year are recorded in the file, i.e. total 200 files at the same time interval. Then according to data in each file, equations (1) were again integrated over a time interval equal to the orbital period of Apophis, and the coordinates and speeds of the asteroid and the sun were stored in the new file. According to these data, using the DefTra program, parameters of the Apophis orbit relative to the Sun were determined in the equatorial coordinate system. Such calculations were carried out for each of the 200 files. They are conducted in an automated mode under control of the PaOrb program. Thereafter angular orbital parameters have been translated into the ecliptic coordinate system (see Fig. 2).

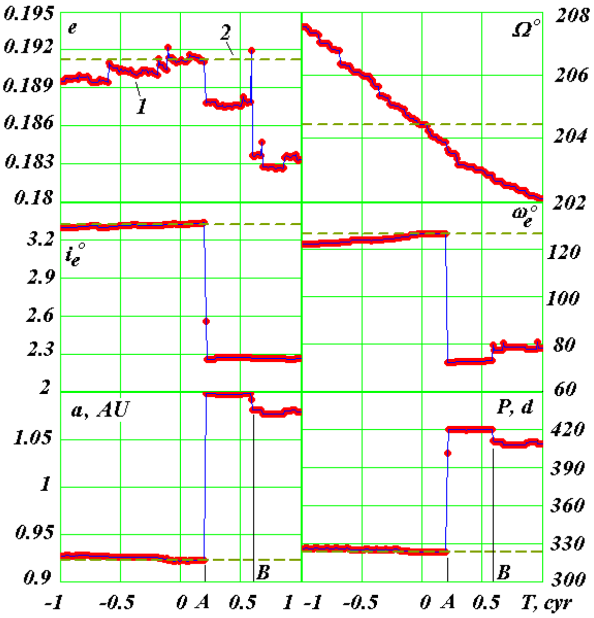


Fig. 2. Evolution of the parameters of the Apophis orbit under the influence of planets, moon and sun on the range -100 ÷ 100 years from November 30, 2008: 1 – per the results of the integration of the motion equations (1), 2 - initial values according to table 1. Angular values: Ω, i_e, ω_e - are given in degrees, semi-major axis a - in the a.u. and orbital period P - in days.

As can be seen from Fig. 2, the eccentricity e of the Apophis orbit varies irregularly. There are jumps or breaks of eccentricity. One of the significant gaps is observed at the time T_A, when Apophis approaches to the Earth on the shortest distance. Second Significant leap of eccentricity occurs in approach to Earth at time T_B. Longitude Ω of ascending node is less prone to breaks and, as seen from Fig. 2, decreases almost monotonously. The remaining elements of the orbit i_e, ω_e, a and P have significant gaps in time (T_A) of the closest passage of Apophis by Earth.

In the graphs of Fig. 2 dashed line shows values of the orbital elements of the initial time, which are presented in table 1. As can be seen from the graphs, they coincide with the orbital elements obtained by integrating the equations (1), in the moment $T = 0$: The relative difference between the parameters e, Ω, i_e, ω_e, a and P and the initial values (table 1) is $9.4 \cdot 10^{-6}$, $-1.1 \cdot 10^{-6}$, $3.7 \cdot 10^{-6}$, $-8.5 \cdot 10^{-6}$, $1.7 \cdot 10^{-5}$ and $3.1 \cdot 10^{-5}$ respectively. This coincidence indicates the reliability of calculations performed at all stages: determination of the initial conditions, integration of the equations, orbit parameters determination and conversion between different coordinate systems

In addition to simplified differential equations (1) of motion of celestial bodies, we use other equations, as mentioned in the introduction. As is well known [20], the orbital elements are used in the equations of perturbed motion. Therefore, these equations will give significant errors in the breaks of the orbital parameters presented in Fig. 2. There are also other methods for solving differential equations, including series expansion of per orbit elements, or by using divided differences. As noted in the introduction, they are sensitive to various resonance phenomena and dramatic changes in the orbit in case of the approach of bodies to each other. Equations (1) integrated by us and the used method (2) do not have these disadvantages. This suggests that the results obtained in the present study will not undergo significant changes in the future.

6. Influence of initial conditions.

In order to verify the effect of initial conditions (IC) on the Aphophis trajectory, equations (1) were integrated in the interval of 100 years with two more options of the initial conditions. The second option of IC is set for January 4, 2010 (see table 1). They are taken from the base of small planets [18] and correspond to the solution with the number *JPL* sol. 144 obtained by Steven R. Chesley on October 23, 2009. Fig. 3 shows the results of two solutions with different IC. Line 1 shows the time variation of the distance R between Apophis and the Earth for 100 years with the first option of IC. As can be seen from the graph, the distance R changes in an oscillatory way, and two periods can be distinguished: short period T_{R1}=0.87 years and a long period ofT_{R2}. The amplitude of the short period R_{a1} = 29.3 million km, and of the long one: R_{a2} = 117.6 million km. The value of the long period up to $T \sim 70$ is equal to T_{R20}=7.8 years, and further increases slightly. After an approach on April 13, 2029 (point A in Fig. 3) the amplitude of the second oscillations slightly increases. Both short and long oscillations are not regular, so the above are their average characteristics.

Let's also Note the second (in terms of minimum distance)approach of Apophis to the Earth in the interval of 100 years. It occurs when the T_{F1}=58.37 years (point F1 in Fig. 3) at a distance of R_{F1}=622 thousand km. On April, 13, 2036 (point H in Fig. 3) Apophis passes by the Earth at a distance of R_{H1} =86 million km. The above-mentioned characteristics of the solution are presented in table 4.

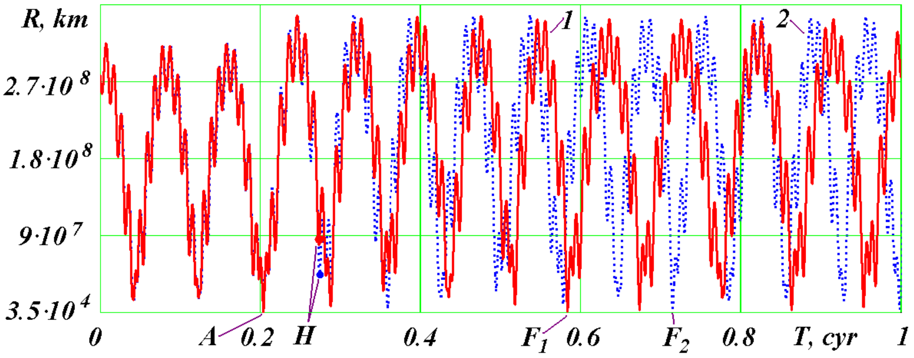


Figure 3. Evolution of the distance R between Apophis and the Earth for 100 years. Influence of initial conditions (IC): 1 - IC on November, 30, 2008, 2 - IC on January, 4, 2010. Calendar dates of approaches in points: A - April 13, 2029; F_1 -April 13, 2067; F_2 -April 14, 2080.

Line 2 is the solution with the second option of IC with integration step dT=$1 \cdot 10^{-5}$ years. The approach time coincided with an accuracy of 1 min, and the distance of approach with second IC became R_{A2} = 37886 km, i.e. it decreased by 1021 km. To clarify these parameters, equations (1) near the point of approach were integrated with step dT=$1 \cdot 10^{-6}$ years. According to updated calculations, Apophis will approach to the Earth at 21 hours 44 minutes 53 seconds at a distance R_{A2}=37880 km. As seen from table 4, this point of approach is different from the moment of rapprochement for the first IC by 8 sec. Since at step $dT = 1\cdot 10^{-6}$ years the accuracy of the time determination is 16 seconds, then times of approach within the accuracy of their calculations coincide.

Short and long oscillations in two versions of IC also coincided until approach. After approach in point A, long oscillation period has decreased to T_{R22}=7.15 years, i.e. become smaller than the period T_{R20} in the first variant of IC. The second approach on the interval of 100 years occurs at T_{F2}=70.28 years at a distance of R_{F2} =1.663 million km. In 2036 (point H) Apophis passes at a distance of R_{H2}=43.8 million km.

In the second option of the initial conditions, IC of Apophis and influencing bodies change on January,4, 2010 as compared with the first option. To reveal the influence of errors of only Apophis's IC , the third option of IC is specified (see table 1) as the first one, on November, 30, 2008, but IC for Apophis are computed in the Horizons system in accordance with the solution JPL sol. 144. As shown in table 1, of the six orbital elements e, a, i_e, Ω, ω_e and M, differences of three: i_e, Ω and ω_e from similar elements of the first option of IC is 2.9, 1.6 and 1.5 of the associated uncertainties σ. Differences of the other elements do not exceed their uncertainties.

Table 4. Influence of initial conditions on the results of the integration of equations (1) by Galactica and the equations of motion of Apophis by Horizons: TimeA and RminA - time and distance of approach of Apophis to Earth on April 13, 2029, respectively ; RH - distance at which Apophis passes by the Earth on April 13, 2036 Mr..; TF and RF - time and distance of the second approach (i.e. point F in Fig. 3).

Parameters	Solutions at different variants of initial conditions.					
	Galactica			Horizons		
	1	2	3	1	2	3
	30.11.2008 JPL sol.140	04.01.2010 JPL sol.144	30.11.2008 JPL sol.144	18.07.2006 JPL sol.144	30.11.2008 JPL sol.140	04.01.2010 JPL sol.144
Time_A	21:44:45	21:44:53	21:44:45	21:46:47	21:45:47	21:44:45
R_{minA}, km	38905	37880	38813	38068	38161	38068
R_H, million of km	86.0	43.8	81.9	51.9	55.9	51.8
T_F, from 30.11.08	0.5837	0.7138	0.6537	0.4237	0.9437	0.4238
R_F, thousand of km	622	1663	585	1515	684	1541

In the third option of IC wih integration step $dT = 1\cdot 10^{-5}$ years, the approach time coincided with that of the first option of IC. The approach distance has became R_{A3} = 38814 km, i.e. it has decreased by 93 km. To clarify these parameters, equations (1) near the point of approach were also integrated with a step $dT = 1\cdot 10^{-6}$ years. According to revised calculations, in the third option of IC Apophis approaches to the Earth at 21 hours 44 minutes 45 seconds at a distance R_{A3} = 38813 km. These and other characteristics of the solution are shown in table 4. In comparison with the first option of IC it's clear that approach distance in 2036 and the parameters of the second convergence in point F_1 slightly change. However, differences between the results of the first option with the third one are much smaller than between the first and the second.

In the second option, the change of positions and velocities of influencing bodies from November 30, 2008 to 04.01.2010 is calculated per DE406, and in the third option – using the program "Galactica". IC for Apophis in two options are defined according to the same solution JPL sol. 144. As seen from table 4, the time of approach in these solutions differs by 8 sec., and approach distance by 933 km. Also, other results of the third solution are different from the second solution in a greater degree in comparison with differences between the third solution and the first one. This indicates that IC differences are less significant for Apophis compared with differences of calculation results from two programs: Galactica and DE406 (or Horizons).

Similar studies on the effect of initial conditions we have conducted with NASA integrator. The Horizons system (the JPL Horizons On-Line Ephemeris System, see online guide http://ssd.jpl.nasa.gov/?horizons_doc) gives an opportunity to calculate the motion of an asteroid based on the same standard dynamic model (SDM), that have been used for calculations in [1]. In addition to two considered IC, we used yet another IC as of July 12, 2006. Features and Highlights of all solutions are presented in Table 4. Approach time on April 13, 2029 changes within 2 minutes and distance is near 38,000 km. Approach distance on April 13, 2036 is between 52 to 56 million km. Characteristics of the second approach in 100 years change approximately in the same range as for the decisions of Galactica. Other above-mentioned laws on the impact of IC also repeated for NASA integrator.

Thus, the calculations for different initial conditions have shown that in 2029 Apophis will approach the Earth at a distance of 38 ÷ 39 thousand km, and in the next 100 years, he will pass by the Earth once more at a distance no closer than 600 thousand kilometers.

7. Study Apophis trajectory when approaching the Earth.

For this purpose, equations (1) were integrated in the interval of two years from T_1 = 0.19 centuries, and the value of the coordinates and velocity of the Earth and Apophis over every 50 integration steps are written to a file. The biennium includes time T_A of the closest passage of Apophis by the Earth. In Fig. 4 ellipse E_0E_1 is a projection of the Earth's trajectory for two years on the equatorial plane xOy. According to it, from the point E_0, the Earth will make two turns. The trajectory of Apophis in the same coordinates for two years is marked with points with letters Ap. Apophis, starting at Ap_0, goes the way and at $Ap_0Ap_1Ap_eAp_2Ap_0Ap_1$ and in Ap_e at T_A approaches the Earth. After the approach it moves in a different orbit, namely $Ap_eAp_3Ap_f$.

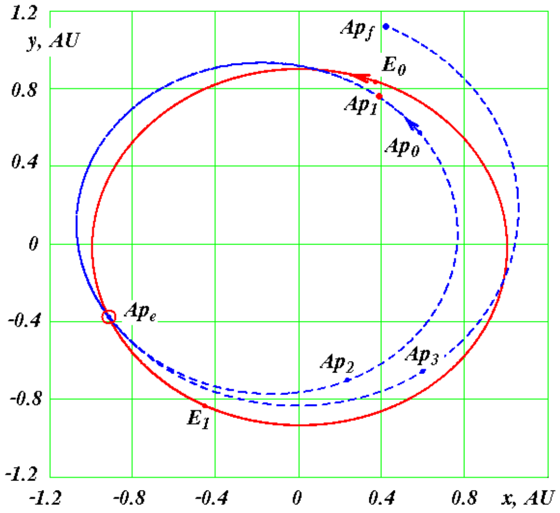


Fig. 4. Apophis (Ap) and Earth (E) trajectories in the equatorial barycentric coordinate system xOy for 2 years: where Ap_0 and E_0 – initial points of Apophis and the Earth; Ap_f – endpoint of Apophis trajectory; Ap_e – point of approach of Apophis to the Earth; the coordinates x and y are given in a.u.

Fig. 5a shows the trajectory of Apophis relative to the Earth. Relative coordinates are defined as the difference between the coordinates of Apophis (Ap) and the Earth (E):

$$y_r = y_{Ap} - y_E; \qquad x_r = x_{Ap} - x_E. \qquad (15)$$

Apophis, starting from the point Ap_0, is moving along the trajectory 1 to the point Ap_e where it approaches the Earth, and its trajectory ends in point Ap_f. Loops on the Apophis trajectory represent its return movements relative to the Earth. All the planets make such loops [21] as seen from the Earth.

At the point of approach Ap_e trajectory of Apophis undergoes a break. This break is shown in Fig. 5b in a large scale. At the origin (point 2) there is the Earth. The Sun (see Fig. 4) is close to the barycenter O, i.e. in the upper right quadrant relative to the point of approach Ap_e.

Therefore Apophis (see Fig. 5b) at the point of approach passes between the Earth and the Sun. As shown below, this fact creates certain difficulties for the use of the asteroid.

8. Possible use of Apophis.

So, on April 13, 2029 there will be a unique phenomenon: the body with weight of 31 million tons will pass by the Earth at a distance of 6 Earth radii from the center of the Earth. In following 1000 years there will be no such approaches of Apophis to the Earth. Many pioneers of space exploration, for example, K.E. Tsiolkovsky, Y. Kondratuk and others imagined space exploration near the Earth using large manned orbital stations. However, to deliver such a large masses from the Earth is a serious technical and environmental problem. Therefore, thanks to a happy occasion, there is the possibility to transform the asteroid Apophis into an Earth satellite, and then into a manned station.

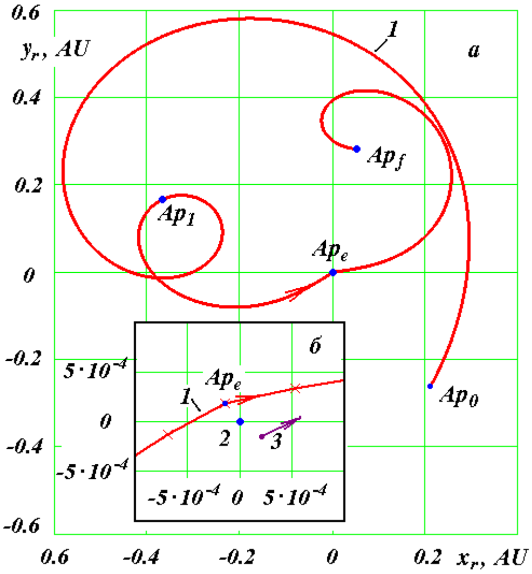


Fig. 5. The trajectory of Apophis (1) in the geocentric equatorial coordinate system x_rOy_r: a – in the normal scale, b - in a larger scale at the time of approach of Apophis to the Earth (2), 3 - position of Apophis at its approach to the Earth after the correction of its trajectory by a factor k = 0.9992 in point Ap_1 in Fig. 4; the coordinates x_r and y_r are given in a.u.

Among the possible various applications of the satellite let's mention two. The first - creation of a space elevator. As is known, the space elevator consists of a rope, one end of it is attached to a point on the Earth's equator, and the other end - to a massive body, which revolves in the equatorial plane with the diurnal rotation period P_d = 24·3600 seconds. The radius of such a geostationary satellite orbit is:

$$R_{gs} = \sqrt[3]{P_d^2 G(m_A + m_E)/4\pi^2} = 42241 \text{ km} = 6.62\, R_{Ee} \qquad (16)$$

To ensure the rope tension, the distance from the center of the Earth to the massive body must be greater than the radius of the geostationary orbit Rgs. Using this rope or several of them, different loads can be derived in space, and other loads can be delivered to the Earth from space.

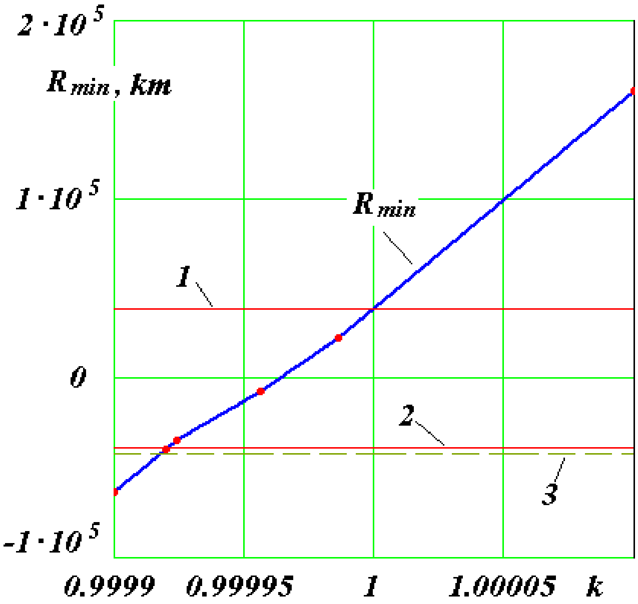
If we turn Apophis into a satellite and then turn that orbit in the equatorial plane, then the satellite can be used to build a space elevator. The Second application is to use the asteroid as a "shuttle" to deliver goods to the moon. In this case, the asteroid should have an elongated orbit with a perigee radius close to the radius of the geostationary orbit and the apogee radius approaching perigee radius of the lunar orbit.

In this case, the loads from the geostationary orbit at perigee would be shifted to Apophis, and then at the apogee these goods could be delivered to the moon. Two presented schemes of using the asteroid require overcoming many challenges that may now seem even insolvable. But, of course, it is clear that these problems can't be really solved if Apophis is not turned into a satellite of the Earth. So let's consider what are the opportunities here. Velocity of the asteroid relative to the Earth at the point of approach Ape is equal to v_{AE}=7.39 km/s. Velocity of the Earth satellite at a distance RminA in a circular orbit:

$$v_{CE} = \sqrt{G(m_A + m_E)/R_{\min A}} = 3.2 \text{ km/s} \qquad (17)$$

To turn the asteroid into a satellite it's needed to bring his speed v_{AE} to v_{CE}. Equations (1) were integrated when Apophis at TA had its speed reduced by 1.9 times, i.e, v_{AE} = 7.39 km/s, is reduced to 3.89 km/s. In this case, Apophis turns into a satellite of the Earthwith the following orbital parameters: eccentricity e_{s1} = 0.476; angle to the plane of the equator i_{s1} = 39.2°; semi-major axis a_{s1} = 74540 km and the sidereal period P_{s1} = 2.344 days. We explored the evolution of the satellite motion over 100 years. Despite the more significant oscillations of elements of its orbit compared with the oscillations of elements of the planetary orbits, semi-major axis and orbital period of the satellite are close to the specified values. Their relative changes do not exceed: $|\delta a| < \pm 2.75\cdot10^{-4}$ и $|\delta P| < \pm 4.46\cdot10^{-4}$. However, the satellites rotation is against the Earth's rotation and orbital rotation of the moon. Therefore, the use of such a satellite in two considered cases is impossible.

Thus, the satellite should have the same rotation as the rotation of the Earth. If Apophis (see Fig. 5b) will bend around the Earth not from the day side, as shown by line 1, but from the night side (see point 3), then in case of a decrease of its speed, it will become a satellite that will rotate in the required direction. For this purpose, equations (1) were integrated with speed variation of the asteroid in point Ap_1 in Fig. 4. In this point in the orbit, located approximately at half-rotation from the point Ap_e of approach to the Earth, Apophis is at the moment century T_{Ap1}=0.149263369488169 centuries. In point Ap1, projections of the velocity of Apophis in barycentric equatorial coordinate system are: v_{Ap1x} = -25.6136689 km/s; v_{Ap1y} = 17.75185451 km/s; v_{Ap1z}= 5.95159206 km/s. In these numerical experiments, velocity components were changed proportionally the same number of times, i.e. they were multiplied by a factor *k*, then equations (1) were integrated, and the trajectory of the asteroid was determined. Fig. 6 shows the dependence of the minimum approach of Apophis to the center of the Earth, depending on the factor *k* decreasing its speed in Ap_1.



distance R_{min} at which Apophis passes by the center of the Earth from the correction factor k for its velocity at point Ap_1 in Fig. 4. Positive values of R_{min} correspond to the dayside: R_{min} – in km; 1 – minimum distance Rmin at which Apophis passes by the center of the Earth on April 13, 2029 (the day side), 2 - – minimum distance Rmin at which Apophis passes by the center of the Earth after orbital correction (night side), 3 - the radius of the geostationary orbit R_{gs}.

As a result, it was found that a decrease in the coefficient k (see Fig. 6) the asteroid begins to approach closer to the Earth, and if the factor *k* = 0.9999564, Apophis collides with the Earth. With further decrease of the speed of the asteroid it approaches the Earth from the side opposite to the Sun, and at *k* = 0.9992 the asteroid passes (point 3 in Fig. 5b) at a distance R_{min3} = 39157 km from the center of the Earth at time T_3 = 0.2036882 centruies. Distance R_{min3} is substantially the same as the distance R_{minA} when asteroid passes between the Earth and the Sun.

In this case, the velocity of the asteroid relative to the Earth is also v_{AE}=7.39 km/s. When decreasing is 1.9 times, i.e. to 3.89 km/s, Apophis turns into a satellite of the Earth with the following orbital parameters: eccentricity e_{s2} = 0.486; inclination of the plane of the equator i_{s2} = 36°; semimajor axis a_{s2} = 76480 km and sidereal period P_{s2} = 2.436 days. We also investigated the evolution of the motion of the satellite over 100 years. Satellite orbit is also stable, and he turns in the same direction as the Moon.

So, for the conversion of Apophis into the satellite with the necessary direction of its rotation it's required to do two reductions of its speed. The first is carried out before its approach to the Earth, such as in point Ap1 (Fig. 4) for 0.443 years before the approach of Apophis to the Earth. The velocity of Apophis should be decreased by 2.54 meter/s. Second deceleration of the asteroid must be implemented at the time of approach to the Earth. In our example, its elliptical orbit speed must be reduced by 3.5 km/s.

Reducing the speed of the body with mass of 30 million tons by 3.5 km/s is now a serious scientific and technical problem. For example, in [4] imparting to Apophis the velocity about 10^{-6} meter/s is deemed possible by available technical means. But speed increase by cm/s the authors of [4] already consider as a complex scientific and technical problem. But 20 years are ahead. And we know that after the Second World War, almost in 10 years, a much more serious problem has been solved: creation of the first artificial satellite, and then a manned spacecraft. Therefore, there is no doubt that if the society set this goal, it will be successfully achieved.

It should be noted that in [1], the authors consider the possibility of changing the orbit of Apophis for its collision with asteroid (144898) 2004 VD17. There is a small probability of a collision of the second asteroid with Earth in 2102. However, questions about the necessary precision of movement coordination for both of asteroids at once raise doubts about the probability to solve this problem. This example and others show that many researchers come to the conclusion that significant impacts on the asteroid are required to solve a variety of space problems, including anti-asteroid protection of the Earth. If people will solve this problem of converting an asteroid into a satellite, then the ability to prevent a serious danger from an asteroid will increase many times.

Conclusions.

1. The disadvantages of existing methods of calculating the motion of the asteroid have been analyzed.
2. Using a new method, over unsimplified differential equations of motion of Apophis, planets, moon and sun for 1000 years have been nuberically integrated and the evolution of the asteroid's orbit has been studied.
3. At 21 o'clock 45 ' GMT on April 13, 2029, Apophis passed by the Earth at a distance of 6 Earth radii from the center. It was the closest approach of Apophis to the Earth in the next 1000 years.
4. Calculations of turning Apophis into a satellite that can solve various tasks for future space exploration have been made.

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1. The authors are grateful to T.J. Galushina and V.G. Pol for materials provided on the Apophis asteroid.
2. The authors are grateful to the staff of the Jet Propulsion Laboratory (JPL) in the United States, web-sites of this lab were used to obtain the initial conditions for integration. Edward Bouella’s site [24] helped the authors to understand all features of the data on asteroids and avoid mistakes when using them.
3. O.I. Krotov has participated in the calculations of Apophis motion using the Horizons system. Calculations were performed on a supercomputer of Siberian Supercomputer Center SB RAS.

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